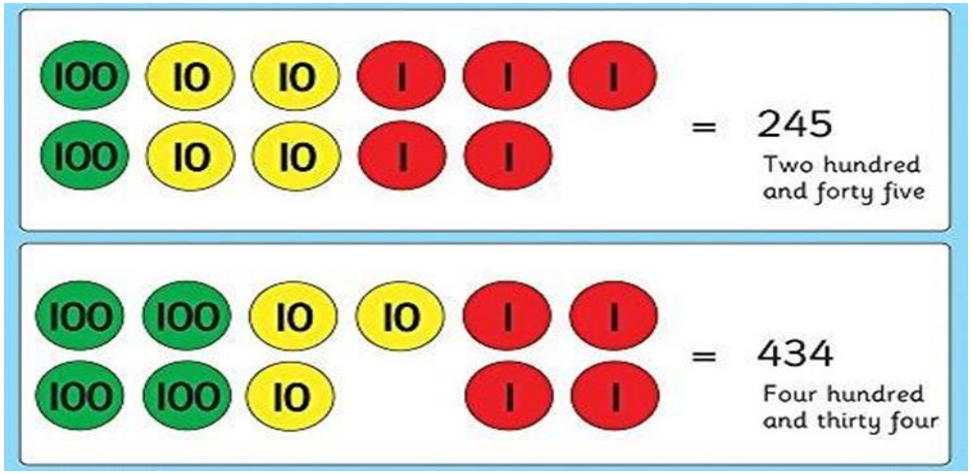
Calculation Policy

Year 3 and Year 4

www.teaguesbridgeprimary.org

Maths Calculation Policy
Year 3 and Year 4

The following pages show our school's progression in calculation (addition, subtraction, multiplication and division) and how this works in line with the National Curriculum. The consistent use of the concrete, pictorial and abstract approach throughout our school helps children develop mastery across all the operations in an efficient and reliable way. This policy shows how these methods develop children's confidence in their understanding of both written and mental methods.



Mathematics Intent

At Teagues Bridge, our intention is **ambitious.** We aim to create strong mathematicians who have the necessary skills and understanding to tackle mathematical challenges in varying contexts, including the ability to reason and apply their knowledge to solving problems. This should mean that children are able to apply their knowledge to everyday life and can **aspire** to achieve anything that they want. We want our pupils to have strong mental manipulation and to use written strategies when appropriate.

Our philosophy for mathematics is replacing an idea that maths is lots of rules and numbers with a study of patterns and connected ideas. In early years they will build a foundation of number understanding and representation through mainly concrete and pictorial representations. The approach will be supported by in depth questioning, throughout the school to develop mastery.

Use of CPA is encouraged to ensure the curriculum is accessible for all children and that they all have the **opportunity** and are able to demonstrate their understanding in a variety of ways. This will enable them to have a good understanding of maths and not just the ability to follow a procedure. We want to **empower** them to want to ask questions and want to find the answers.

Aims: The national curriculum for mathematics aims to ensure that all pupils:

- become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can solve problems by applying their mathematics to a variety of routine and nonroutine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. The programmes of study are, by necessity, organised into apparently distinct domains, but pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects.

The expectation is that the majority of pupils will move through the programmes of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through

being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.

Our lessons are structured to enable all children to achieve and have an **opportunity** to make progress with their learning. Each lesson begins with a **CLIC maths** activity, where they have chance to develop their mental strategies, secure number facts and number manipulation. They then **develop** their mathematical fluency with the teacher modelling and explaining before they have a go themselves. Children then have a **reasoning/ problem solving** activity which is a variation of the previous work to demonstrate they have mastered the objective. Children who are ready can then **challenge** themselves with a task that requires applying the learning to a greater depth. We have our own programme of study which is supported with schemes like White Rose to support.

National Curriculum	Year 3	Known Facts	Essential Knowledge	Year 4	Known facts	Essential Knowledge
Addition	Add and subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction.	Derive and use addition and subtraction facts to 100, e.g. 33 + 67 =100.	Add single digit bridging through boundaries. Partion second number to add. Use near doubles to add. Partion and recombine. Add multiples 10, 100. Pairs of 100 (compliments of 100). Add near multiples of 10 and 100 by rounding and adjusting	Add and subtract numbers with up to 4 digits using the formal written method of columnar addition and subtraction where appropriate. Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.	Derive and use addition and subtraction facts (for multiples of 10) to 1000, e.g. 330+670=1000.	Fluency of 2 digit+ 2 digit. Partion second number to add. Use near doubles to add. Add near multiples. Add multiples of 10, 100 and 1000. Decimal pairs of 10 and 1. Adjust both numbers before adding. Partion and recombine.
Subtraction	Add and subtract numbers with up to 3 digits, using formal written methods of columnar addition	Derive and use addition and subtraction facts to 100, e.g. 33+67=100	Subtract single digit bridging through boundaries. Partion second number to subtract. Differences between.	Add and subtract numbers with up to 4 digits using the formal written method of columnar addition	Derive and use addition and subtraction facts (for multiples of 10), to 1000, e.g. 330+670=1000	Fluency of 2-digit – 2-digit. Partion second number to subtract. Difference between.

	and subtraction. Least significant digit is always dealt with first to establish if the exchange is needed.		Partion and recombine. Subtract multiples of 10, 100. Pairs of 100 (complements of 100). Subtract near multiples of 10 and 100 by rounding and adjusting.	and subtraction where appropriate. Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.		Subtract multiples of 10, 100 and 1000. Decimal subtraction from 10 or 1. Subtract near multiples by rounding and adjusting.
Multiplication	Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including 2 digit numbers times 1 digit numbers progressing to formal written methods.	Recall and use x and ÷ facts for the 3, 4 and 8x tables.	Review 2x, 5x and 10x, 4x, 8x, 3x, 6x. Double 2-digit numbers.	Multiply 2 digit and 3 digit numbers by a 1 digit number using formal written layout. Solve problems involving multiplying and adding.	Recall and use x and ÷ for x tables up to 12 x 12.	4x and 8x tables. 3x, 6x and 12x tables. 3x and 9x tables. 10x bigger, 100x bigger. Double larger numbers and decimals. 11x and 7x tables

Division	Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including 2 digit numbers times 1 digit numbers progressing to formal written methods.	Recall and use x and ÷ facts for the 3, 4 and 8x tables.	Review division facts (2x, 5x and 10x tables). Division facts (4x table). Division facts (8x table). ***Test of divisibility*** ~ KS1 ~ 2, 5 and 10. Halve 2-digit numbers. Division facts (3x table). Division facts (6x table) ***Test of divisibility*** ~ Any number with a digit sum of a multiple of 3, will divide equally by 3.	Practise to become fluent in the formal written method of short division with exact answers.	Recall x and ÷ for x tables up to 12 x 12	Division facts (4x and 8x tables). Division facts (3x, 6x and 12x tables). Division facts (3x and 9x tables). ***Test of divisibility*** ~ Any number with a digit sum of a multiple of 3, will divide equally by 3. KS1 2, 5, 10. 10x smaller. Halve larger numbers and decimals. Division facts (11x and 7x tables). ***Test of divisibility*** ~ Any number with a digit sum of a multiple of 3 and is even will divide equally by 6.

Key Language	Year 3	Year 4
Addition	**Subject specific:** put together, add, addition, altogether, double, total, more than, equals, plus, make, altogether, commutative, inverse, sum, partition, double, near double, one moew, two more.. one hundred more…, hoe many more to make..? How many more is..? how much more is..?**score** **Instructional vocabulary:** explain your method explain how you got your answer give an example of… show how you… show your working…. Estimate	**Subject specific:** put together, add, altogether, double, total, more than, equals, plus, make, commutative, inverse, sum, partition, near double, score, double, near double, how many more to make..? **increase** **Instructional vocabulary:** calculate, work out, solve investigate, question answer check
Subtraction	**Subject specific:** subtract, subtraction, takeaway, distance between, difference between, less than, minus, leave, fewer, left over, how many fewer is…? how much less is…? difference between…? half, halve, equals, tens boundary, partition, rearrange, inverse, **hundreds boundary, exchange, carried digits** **Instructional vocabulary:** explain your method explain how you got your answer give an example of… show how you… show your working	**Subject specific:** subtract, subtraction, takeaway, distance between, difference between, less than, minus, leave, fewer, left over, equals, tens boundary, partition, rearrange, inverse, hundreds boundary, exchange, carried digits, **decrease** **Instructional vocabulary:** calculate, work out, solve investigate, question answer check
Multiplication	Subject specific: double, equal groups, array, lots of, odd, even, commutative, repeated addition, inverse, groups of, multiply, multiplied by, multiple of, twice, row, column, repeated addition, array row, column double, halve share, share equally, one each, two each **tables, factor, related fact, scale, product** **Instructional vocabulary:** carry on, continue repeat what comes next? predict describe the pattern, describe the rule find, find all, find different, investigate choose, decide, collect	**Subject specific:** double, equal groups, array, lots of, odd, even, commutative, repeated addition, inverse, groups of, multiply, multiplied by, multiple of, twice, row, column, tables, factor, related fact, scale, product, repeated addition, array row, column double, halve, **factor pair, known fact, derived fact** **Instructional vocabulary:** carry on, continue, repeat what comes next? predict describe the pattern, describe the rule pattern, puzzle, calculate, calculation, mental calculation, method, jotting, answer right, correct, wrong what could we try next? how did you work it out? number sentence sign, operation, symbol, equation
Division	**Subject specific:**	**Subject specific:**

	share, share equally, equal groups, array, pairs, divide, divided by, divided into, left over, odd, even, repeated addition, group in pairs, threes, ten equal groups, **remainder, dividend, divisor** **Instructional vocabulary:** calculate, work out, solve, investigate question, answer, check	share, share equally, one each, two each, three each, equal groups, group in pairs, threes, ten equal groups, array, pairs, divide, divided by, divided into, left over, odd, even, repeated addition, remainder, dividend, divisor **Instructional vocabulary:** calculate, work out, solve, investigate, question, answer, check

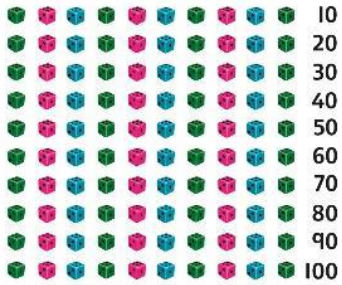
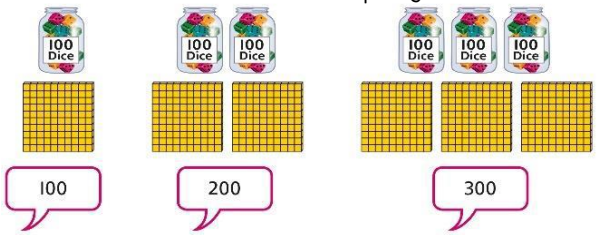
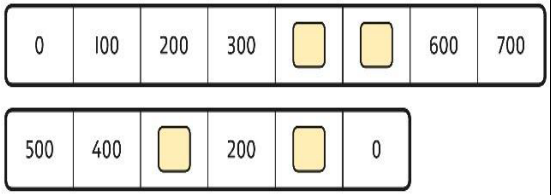
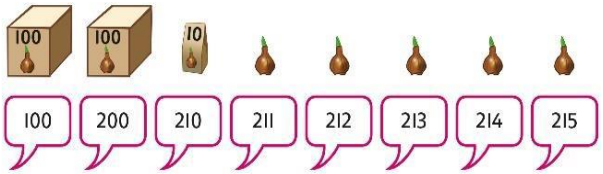
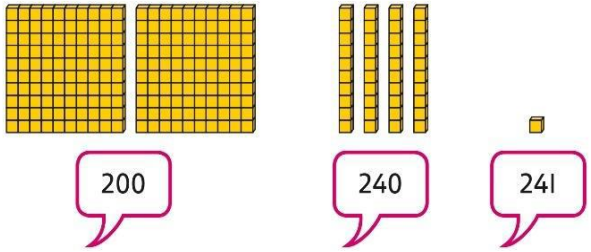
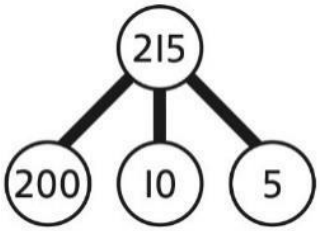
KEYSTAGE 2

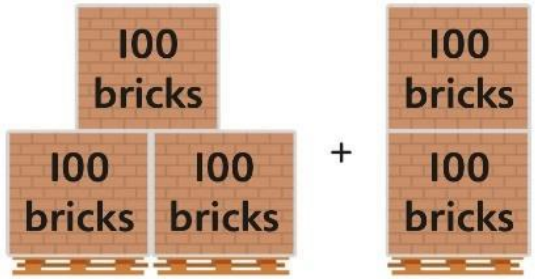
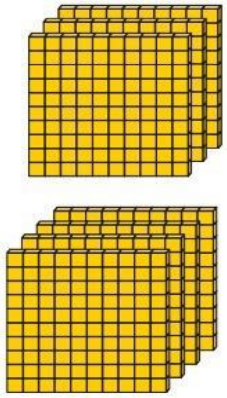
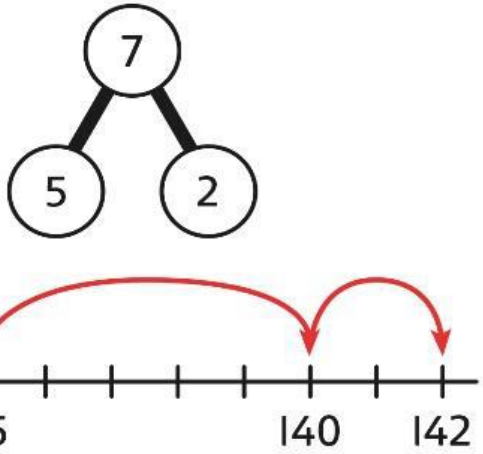
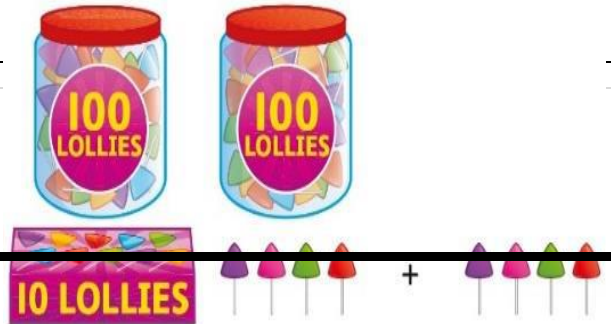
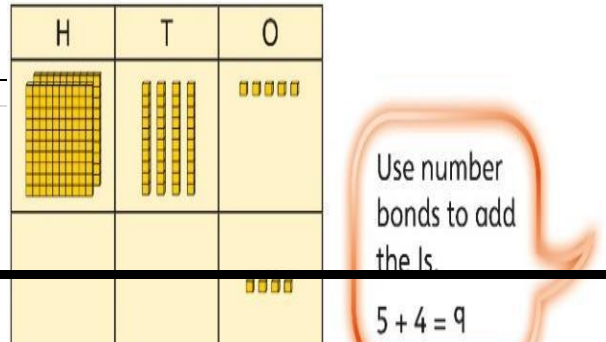
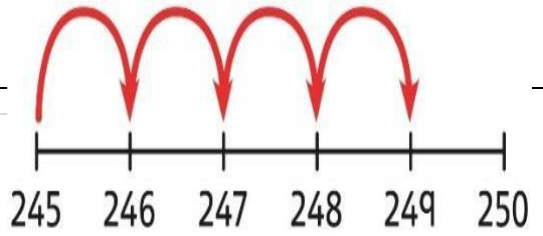
In Years 3 and 4, children develop the basis of written methods by building their skills alongside a deep understanding of place value. They should use known addition/subtraction and multiplication/division facts to calculate efficiently and accurately, rather than relying on counting. Children use place value equipment to support their understanding, but not as a substitute for thinking.

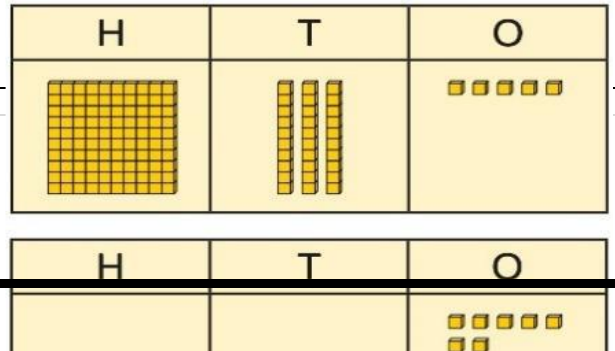
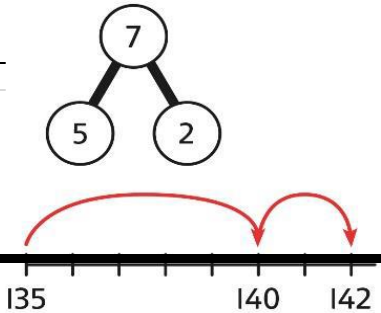
Addition and Subtraction	Multiplication and Division	Fractions
In Year 3 especially, the column methods are built up gradually. Children will develop their understanding of how each stage of the calculation, including any exchanges, relates to place value. The example calculations chosen to introduce the stages of each method may often be more suited to a mental method. However, the examples and the progression of the steps have been chosen to help children develop their fluency in the process, alongside a deep understanding of the concepts and the numbers involved, so that they can apply these skills accurately and efficiently to later calculations. The class should be encouraged to compare mental and written methods for specific calculations, and children should be encouraged at every stage to make choices about which methods to apply.	Children build a solid grounding in times-tables, understanding the multiplication and division facts in tandem. As such, they should be as confident knowing that 35 divided by 7 is 5 as knowing that 5 times 7 is 35. Children develop key skills to support multiplication methods: unitising, commutativity, and how to use partitioning effectively. Unitising allows children to use known facts to multiply and divide multiples of 10 and 100 efficiently. Commutativity gives children flexibility in applying known facts to calculations and problem solving. An understanding of partitioning allows children to extend their skills to multiplying and dividing 2- and 3-digit numbers by a single digit.	Children develop the key concept of equivalent fractions, and link this with multiplying and dividing the numerators and denominators, as well as exploring the visual concept through fractions of shapes. Children learn how to find a fraction of an amount and develop this with the aid of a bar model and other representations alongside. in Year 3, children develop an understanding of how to add and subtract fractions with the same denominator and find complements to the whole. This is developed alongside an understanding of fractions as numbers, including fractions greater than 1. In Year 4, children begin to work with fractions greater than 1.

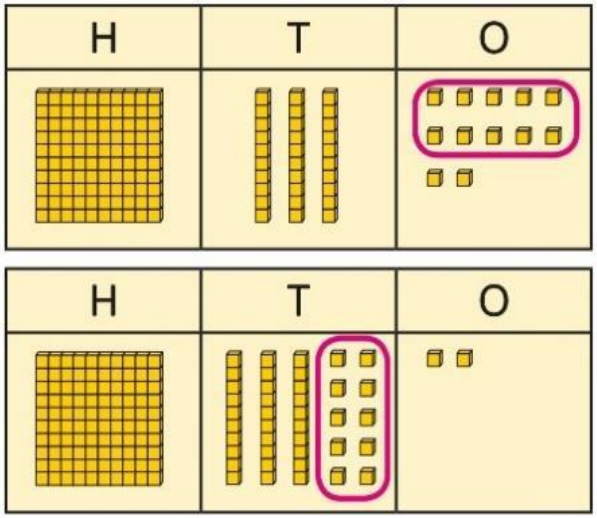
In Year 4, the steps are shown without such fine detail, although children should continue to build their understanding with a secure basis in place value. In subtraction, children will need to develop their understanding of exchange as they may need to exchange across one or two columns. By the end of Year 4, children should have developed fluency in column methods alongside a deep understanding, which will allow them to progress confidently in upper Key Stage 2.	Children develop column methods to support multiplications in these cases. For successful division, children will need to make choices about how to partition. For example, to divide 423 by 3, it is effective to partition 423 into 300, 120 and 3, as these can be divided by 3 using known facts. Children will also need to understand the concept of remainder, in terms of a given calculation and in terms of the context of the problem.	Decimals are introduced, as tenths in Year 3 and then as hundredths in Year 4. Children develop an understanding of decimals in terms of the relationship with fractions, with dividing by 10 and 100, and also with place value.

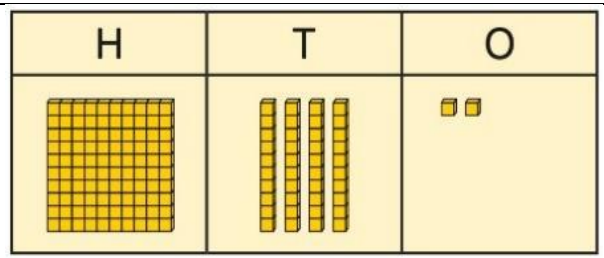
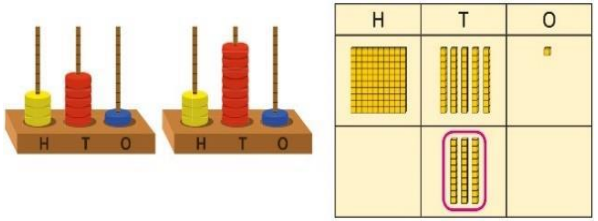
YEAR 3

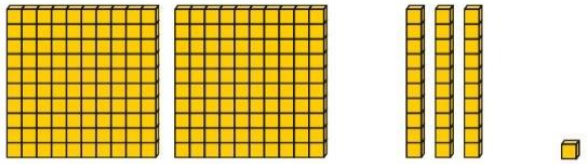
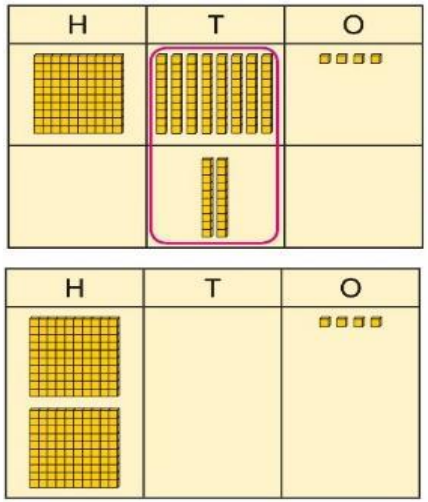
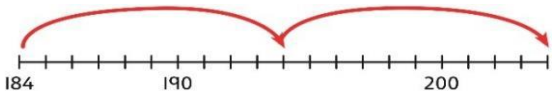
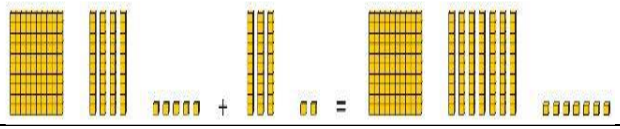
	Concrete	Pictorial	Abstract
YEAR 3 Addition			
Understanding 100s	Understand the cardinality of 100, and the link with 10 tens. Use cubes to place into groups of 10 tens. 	Unitise 100 and count in steps of 100. 	Represent steps of 100 on a number line and a number track and count up to 1,000 and back to 0. 
Understanding place value to 1,000	Unitise 100s, 10s and 1s to build 3-digit numbers. 	Use equipment to represent numbers to 1,000.  Use a place value grid to support the structure of numbers to 1,000. Place value counters are used alongside other equipment. Children should understand how	Represent the parts of numbers to 1,000 using a part-whole model.  *215 = 200 + 10 + 5* Recognise numbers to 1,000 represented on a number line, including those between intervals.

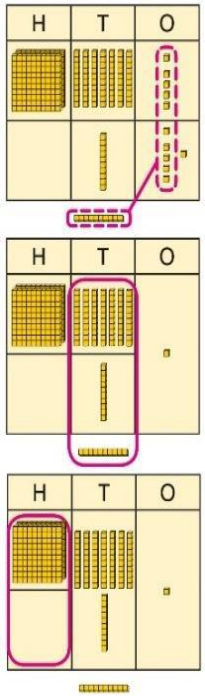
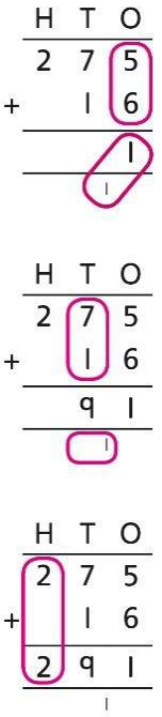
		each counter represents a different unitised amount.	
Adding 100s	Use known facts and unitising to add multiples of 100.  *3 + 2 = 5* *3 hundreds + 2 hundreds = 5 hundreds* *300 + 200 = 500*	Use known facts and unitising to add multiples of 100.  *3 + 4 = 7* *3 hundreds + 4 hundreds = 7 hundreds* *300 + 400 = 700*	Use known facts and unitising to add multiples of 100. Represent the addition on a number line. Use a part-whole model to support unitising.  *3 + 2 = 5* *300 + 200 = 500*
3-digit number + 1s, no exchange or bridging.	Use number bonds to add the 1s. 	Use number bonds to add 1s. 	Understand the link with counting on 245 + 4 

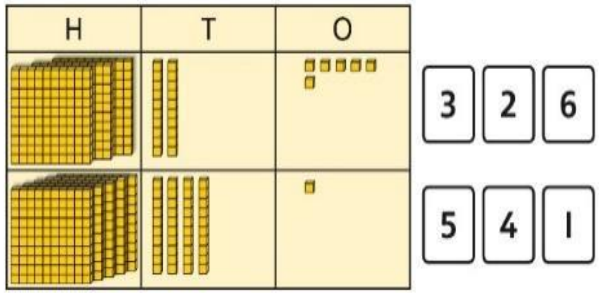
	214 + 4 = ? Now there are 4 + 4 ones in total. 4 + 4 = 8 214 + 4 = 218	245 + 4 5 + 4 = 9 245 + 4 = 249	Use number bonds to add the 1s and understand that this is more efficient and less prone to error. 245 + 4 = ? I will add the 1s. 5 + 4 = 9 So, 245 + 4 = 249
3-digit number + 1s with exchange	Understand that when the 1s sum to 10 or more, this requires an exchange of 10 ones for 1 ten. Children should explore this using unitised objects or physical apparatus.	Exchange 10 ones for 1 te where needed. Use a place value grid to support the understanding. 	Understand how to bridge by partioning to the 1s to make the next 10. 

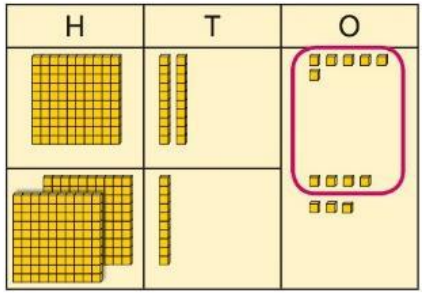
			*135 + 7 = ?* *135 + 5 + 2 = 142* Ensure that children understand how to add 1s bridging a 100. *198 + 5 = ?* *198 + 2 + 3 = 203*

		 135 + 7 = 142	
3-digit number + 10s, no exchange	Calculate mentally by forming the number bond for the 10s.  *234 + 50* *There are 3 tens and 5 tens altogether.* *3 + 5 = 8*	Calculate mentally by forming the number bond for the 10s. *351 + 30 = ?*  *5 tens + 3 tens = 8 tens* *351 + 30 = 381*	Calculate mentally by forming the number bond for the 10s. *753 + 40* *I know that 5 + 4 = 9* *So, 50 + 40 = 90* 753 + 40 = 793

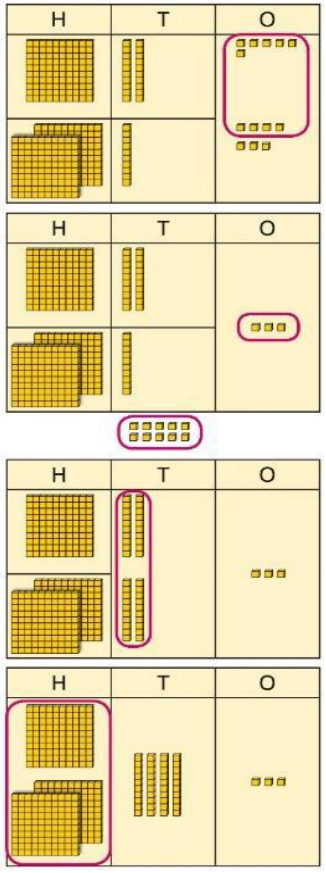
	In total there are 8 tens. *234 + 50 = 284*		
	Understand the exchange of 10 tens for 1 hundred. 	Add by exchanging 10 tens for 1 hundred. *184 + 20 = ?*  *184 + 20 = 204*	Understand how the addition relates to counting on in 10s across 100.  *184 + 20 = ?* *I can count in 10s … 194 … 204* *184 + 20 = 204* Use number bonds within 20 to support efficient mental calculations. *385 + 50* *There are 8 tens and 5 tens.* *That is 13 tens.* *385 + 50 = 300 + 130 + 5* *385 + 50 = 435*
3-digit number + 2-digit number	Use place value equipment to make and combine groups to model addition. 	Use a place value grid to organise thinking and adding of 1s, then 10s.	Use the vertical column method to represent the addition. Children must understand how this relates to place value at each stage of the calculation.

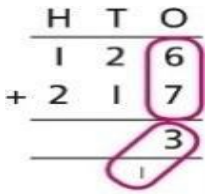
3-digit number + 2-digit number, exchange required	Use place value equipment to model addition and understand where exchange is required. *Use place value counters to represent 154 + 72.* *Use this to decide if any exchange is required.* *There are 5 tens and 7 tens. That is 12 tens so I will exchange.*	Represent the required exchange on a place value grid using equipment. *275 + 16 = ?*  *275 + 16 = 291* Note: In this example, a mental method may be more efficient. The numbers for the	Use a column method with exchange. Children must understand how the method relates to place value at each stage of the calculation.  *275 + 16 = 291*

		example calculation have been chosen to allow children to visualise the concept and see how the method relates to place value. Children should be encouraged at every stage to select methods that are accurate and efficient.	
3-digit number + 3-digit number, no exchange	Use place value equipment to make a representation of a calculation. This may or may not be structured in a place value grid. *326 + 541 is represented as:* 	Represent the place value grid with equipment to model the stages of column addition.	Use a column method to solve efficiently, using known bonds. Children must understand how this relates to place value at every stage of the calculation.
3-digit number + 3-digit number, exchange required	Use place value equipment to enact the exchange required.	Model the stages of column addition using place value equipment on a place value grid.	Use column addition, ensuring understanding of place value at every stage of the calculation.

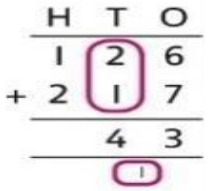


There are 13 ones.
I will exchange 10 ones for 1 ten.

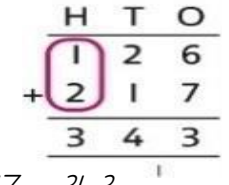




H	T	O
1	2	6
+ 2	1	7
		3
	1	



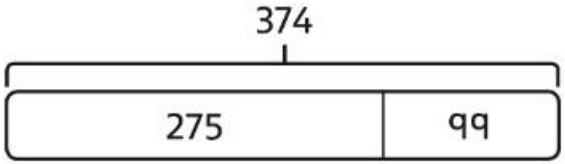
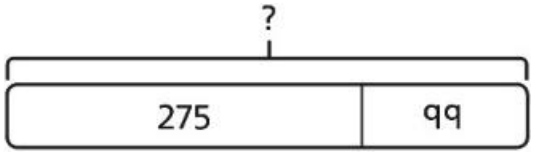
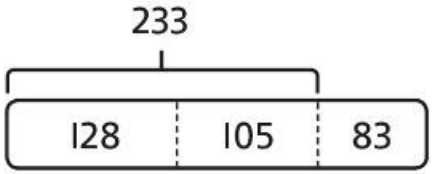
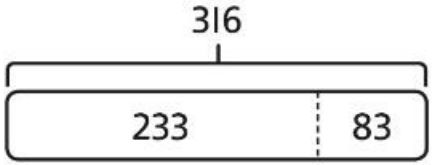
H	T	O
1	2	6
+ 2	1	7
	4	3
	1	

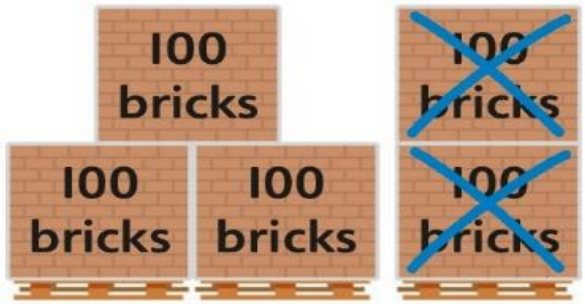
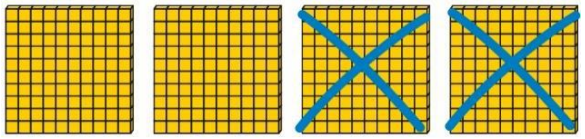
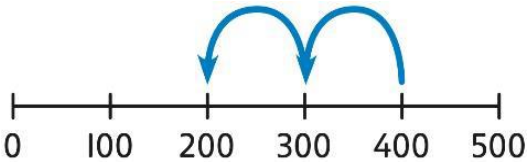
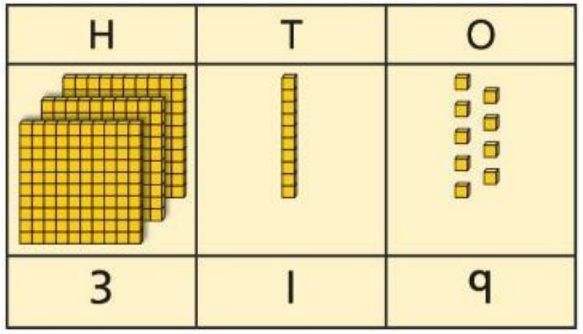
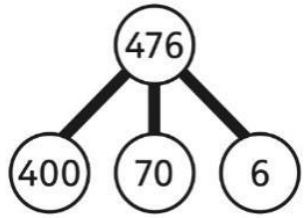


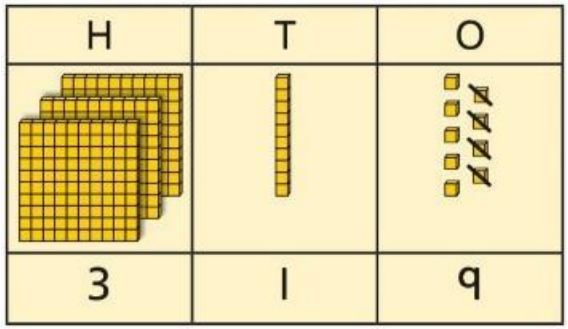
H	T	O
1	2	6
+ 2	1	7
3	4	3
	1	

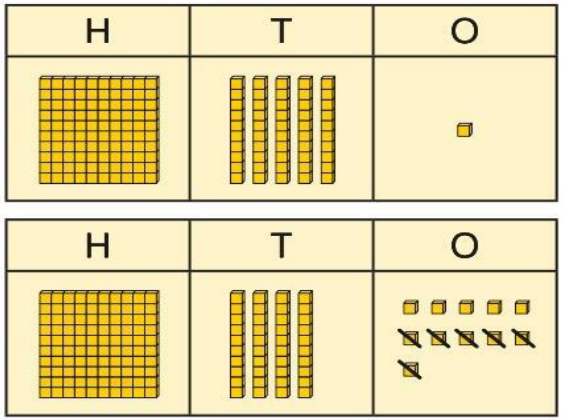
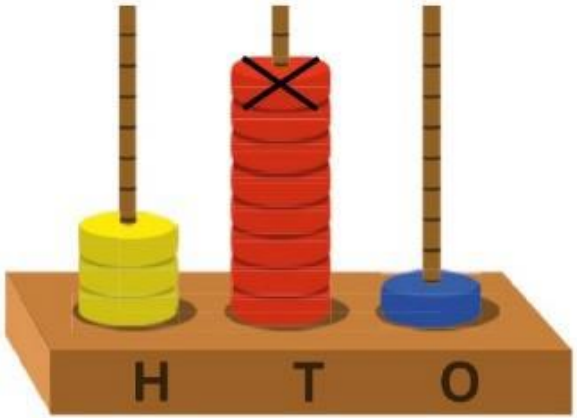
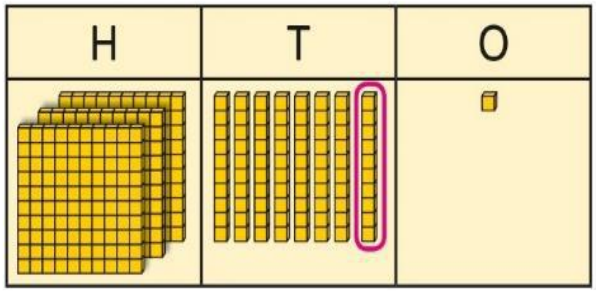
126 + 217 = 343

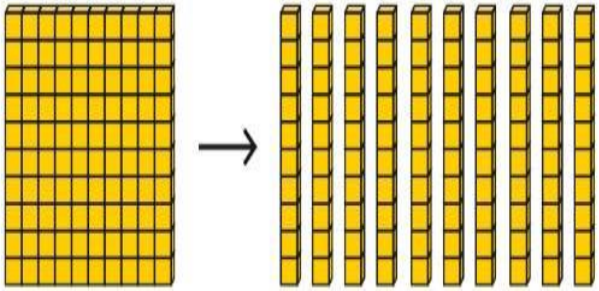
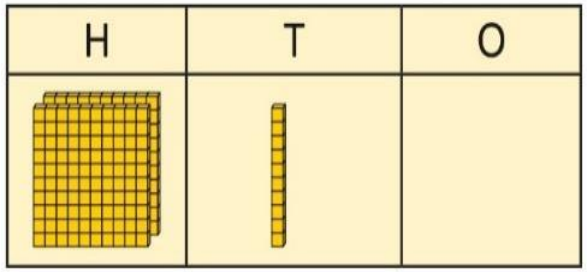
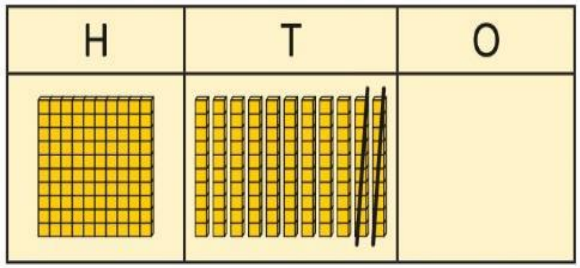
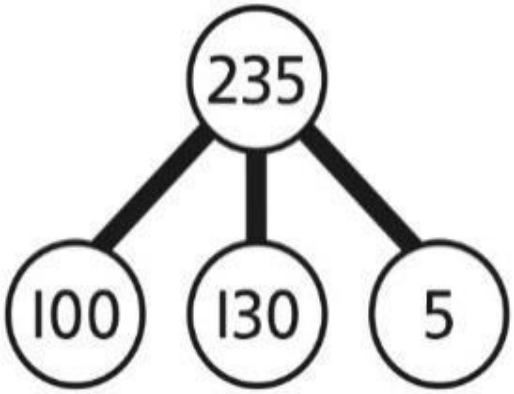
Note: Children should also study examples where exchange is required in more than

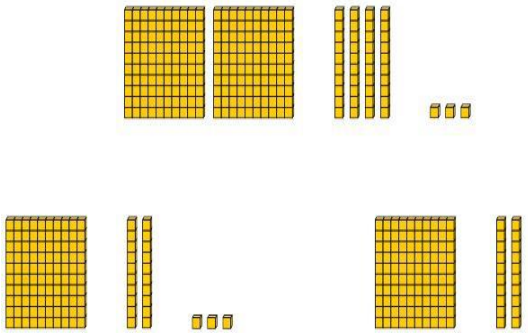
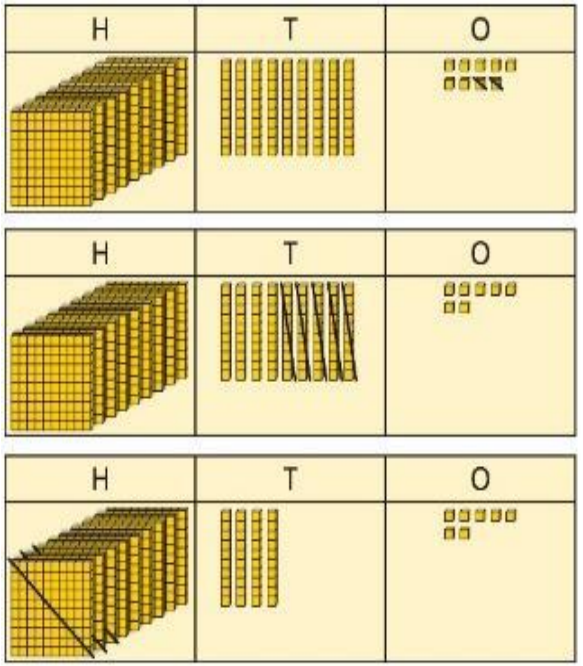
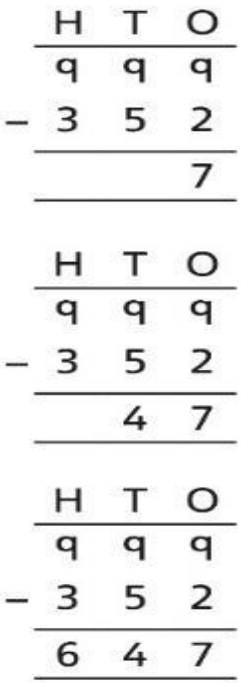
Representing addition problems, and selecting appropriate methods	Encourage children to use their own drawings and choices of place value equipment to represent problems with one or more steps. These representations will help them to select appropriate methods.	Children understand and create bar models to represent addition problems. *275 + 99 = ?*  *275 + 99 = 374*	Use representations to support choices of appropriate methods.  *I will add 100, then subtract 1 to find the solution.* *128 + 105 + 83 = ?* *I need to add three numbers.* 128 + 105 = 233  

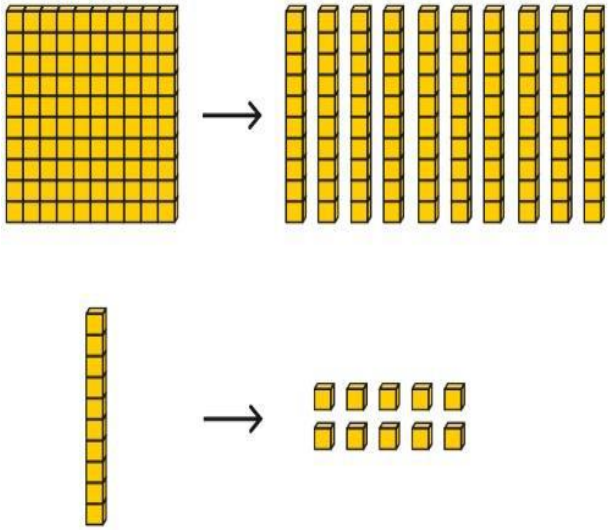
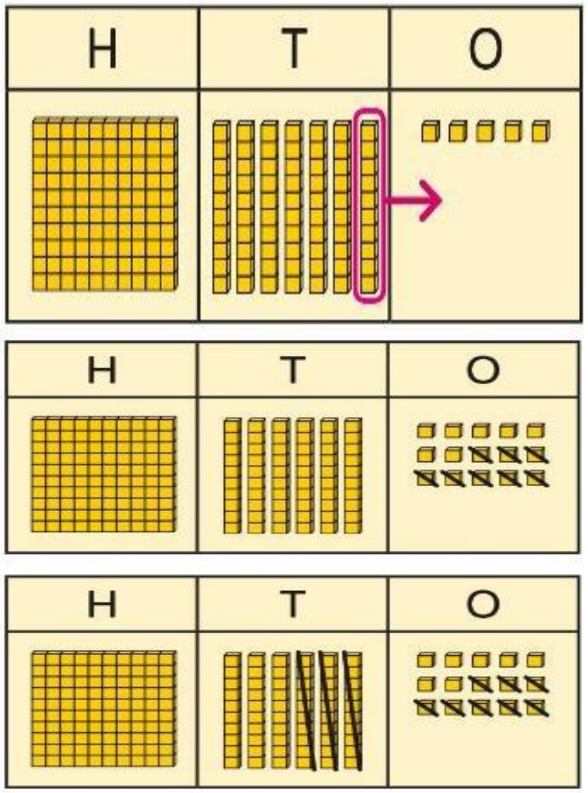
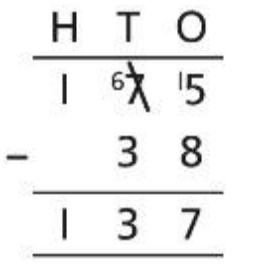
Year 3 Subtraction			
Subtracting 100s	Use known facts and unitising to subtract multiples of 100.  *5 – 2 = 3* *500 – 200 = 300*	Use known facts and unitising to subtract multiples of 100.  *4 – 2 = 2* *400 – 200 = 200*	Understand the link with counting back in 100s.  *400 – 200 = 200* Use known facts and unitising as efficient and accurate methods. *I know that 7 – 4 = 3. Therefore, I know that 700 – 400 = 300.*
3-digit number – 1s, no exchange	Use number bonds to subtract the 1s.  *214 – 3 = ?*	Use number bonds to subtract the 1s.  	Understand the link with counting back using a number line. Use known number bonds to calculate mentally. *476 – 4 = ?* 

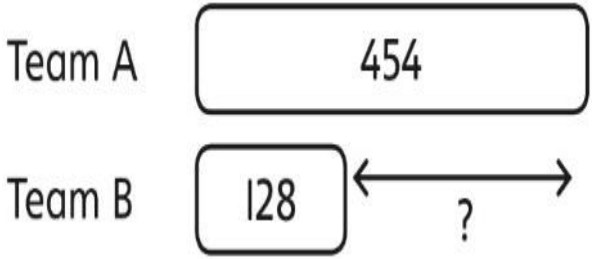
	 *4 – 3 = 1* *214 – 3 = 211*	*319 – 4 = ?*  *9 – 4 = 5* *319 – 4 = 315*	*6 – 4 = 2* *476 – 4 = 472*
3-digit number – 1s, exchange or bridging required	Understand why an exchange is necessary by exploring why 1 ten must be exchanged. Use place value equipment.	Represent the required exchange on a place value grid. *151 – 6 = ?*	Calculate mentally by using known bonds. *151 – 6 = ?* *151 – 1 – 5 = 145*

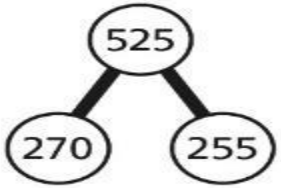
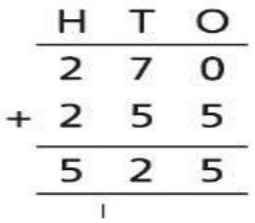
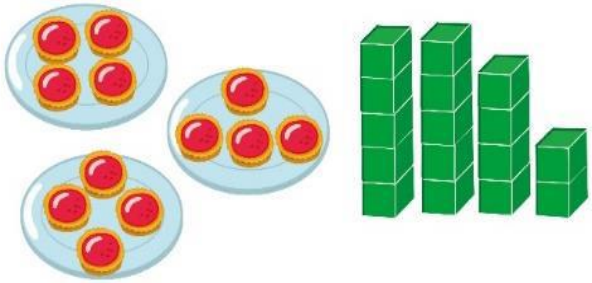
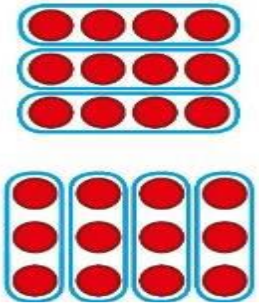
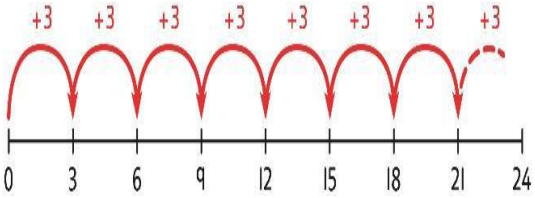
			
3-digit number – 10s, no exchange	Subtract the 10s using known bonds.  *381 – 10 = ?* *8 tens with 1 removed is 7 tens.* *381 – 10 = 371*	Subtract the 10s using known bonds.  *8 tens – 1 ten = 7 tens* *381 – 10 = 371*	Use known bonds to subtract the 10s mentally. *372 – 50 = ?* *70 – 50 = 20* *So, 372 – 50 = 322*

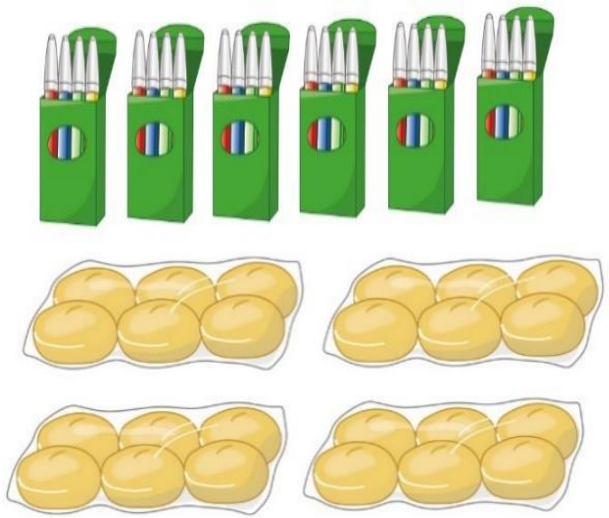
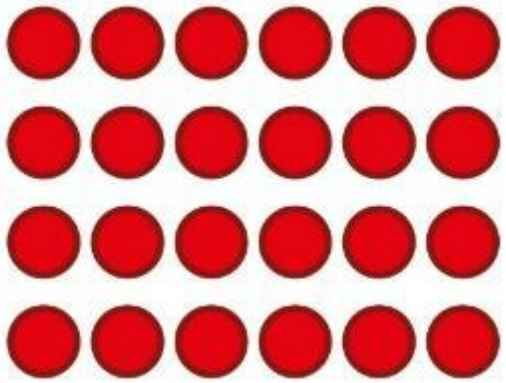
3-digit number – 10s, exchange or bridging required	Use equipment to understand the exchange of 1 hundred for 10 tens. 	Represent the exchange on a place value grid using equipment. *210 – 20 = ?*  *I need to exchange 1 hundred for 10 tens, to help subtract 2 tens.*  *210 – 20 = 190*	Understand the link with counting back on a number line. Use flexible partitioning to support the calculation. *235 – 60 = ?*  235 = 100 + 130 + 5 *235 – 60 = 100 + 70 + 5* = 175

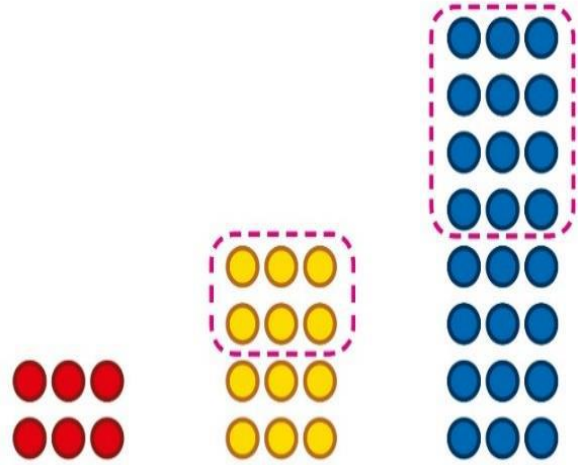
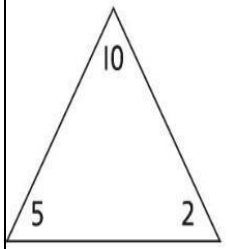
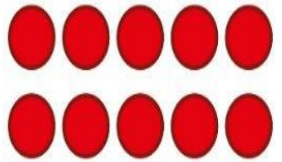
3-digit number – up to 3-digit number	Use place value equipment to explore the effect of splitting a whole into two parts, and understand the link with taking away. 	Represent the calculation on a place value grid. 	Use column subtraction to calculate accurately and efficiently. 

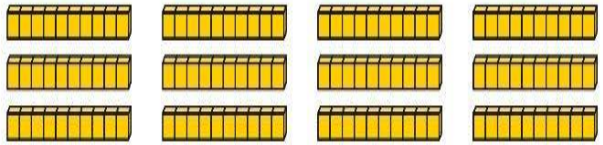
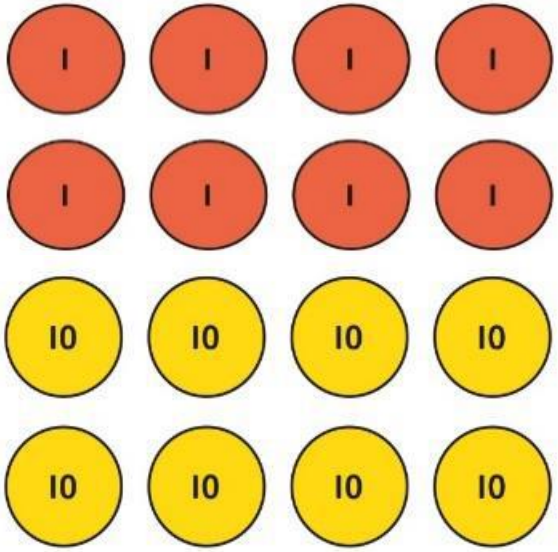
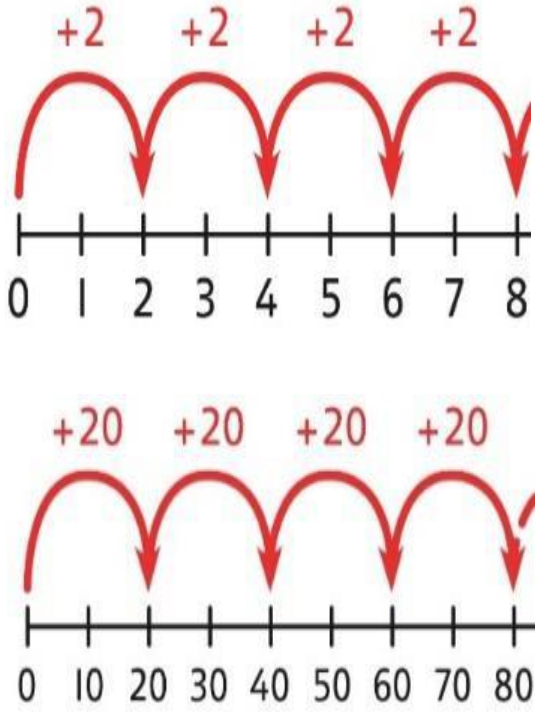
3-digit number – up to 3-digit number, exchange required	Use equipment to enact the exchange of 1 hundred for 10 tens, and 1 ten for 10 ones. 	*Model the required exchange on a place value grid.* *175 – 38 = ?* *I need to subtract 8 ones, so I will exchange a ten for 10 ones.* 	Use column subtraction to work accurately and efficiently.  175 – 38 = 137 If the subtraction is a 3-digit number subtract a 2-digit number, children should understand how the recording relates to the place value, and so how to line up the digits correctly.

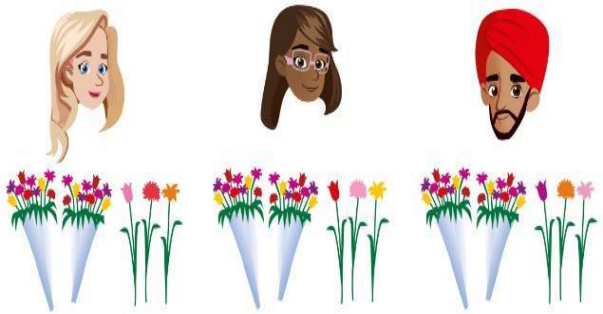
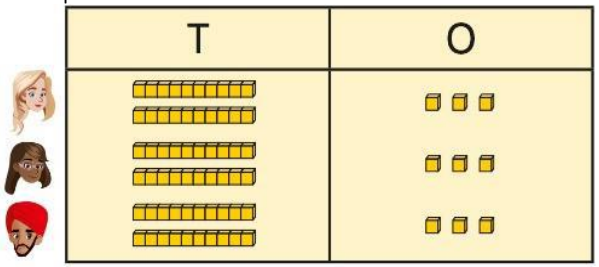
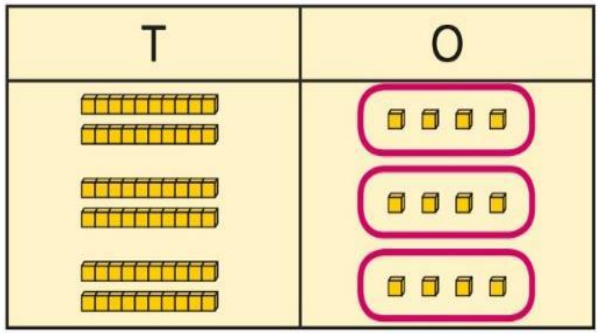
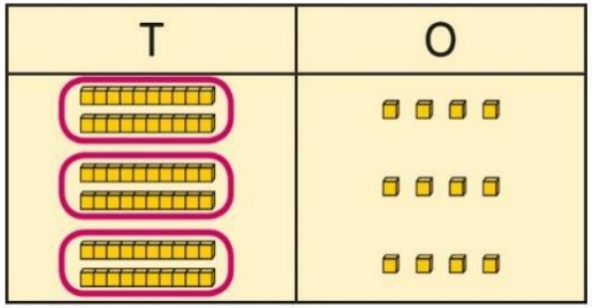
			Children should also understand how to exchange in calculations where there is a zero in the 10s column. 
Representing subtraction problems		Use bar models to represent subtractions. 'Find the difference' is represented as two bars for comparison.  Bar models can also be used to show that a part must be taken away from the whole.	Children use alternative representations to check calculations and choose efficient methods. Children use inverse operations to check additions and subtractions. The part-whole model supports understanding. *I have completed this subtraction.* *525 – 270 = 255* *I will check using addition.*

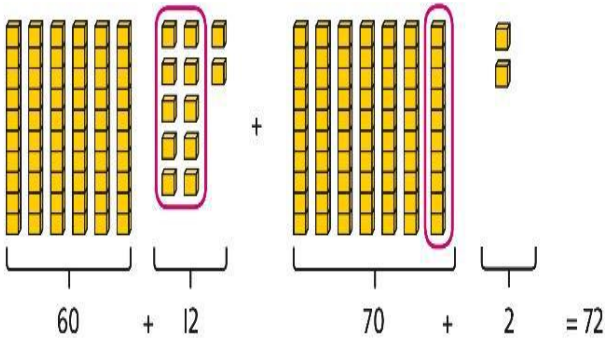
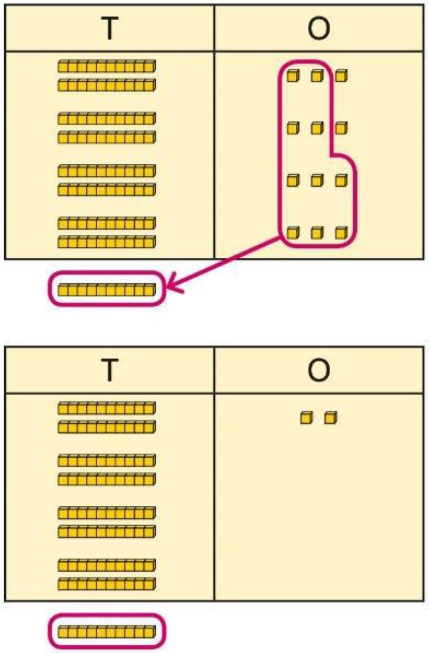
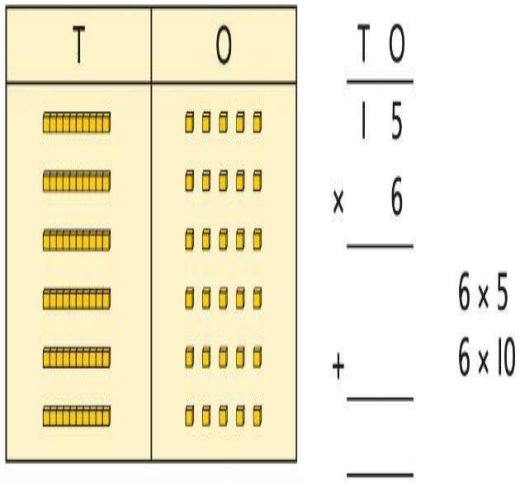
			 
Year 3 Multiplication			
Understanding equal grouping and repeated addition	Children continue to build understanding of equal groups and the relationship with repeated addition. They recognise both examples and non-examples using objects. 	Children recognise that arrays demonstrate commutativity.  *This is 3 groups of 4.* *This is 4 groups of 3.*	Children understand the link between repeated addition and multiplication.  *8 groups of 3 is 24.* *3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 24* *8 × 3 = 24*

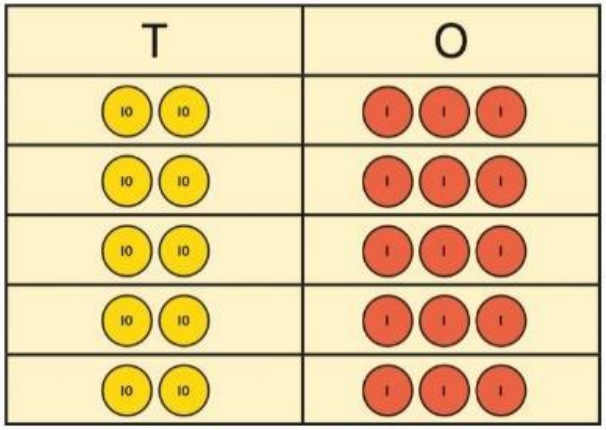
	Children recognise that arrays can be used to model commutative multiplications.  *I can see 3 groups of 8.* *I can see 8 groups of 3.*		A bar model may represent multiplications as equal groups.
Using commutativity to support understanding of the times-tables	Understand how to use times-tables facts flexibly. 	Understand how times-table facts relate to commutativity.  *6 × 4 = 24* *4 × 6 = 24*	Understand how times-table facts relate to commutativity. *I need to work out 4 groups of 7.* *I know that 7 × 4 = 28* *so, I know that* *4 groups of 7 = 28* *and* *7 groups of 4 = 28.*

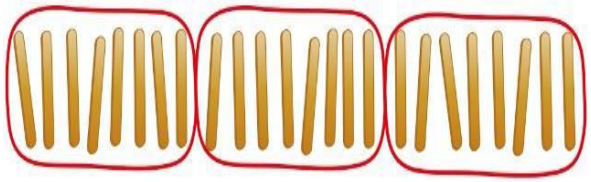
	There are 6 groups of 4 pens. *There are 4 groups of 6 bread rolls.* *I can use 6 × 4 = 24 to work out both totals.*		
Understanding and using ×3, ×2, ×4 and ×8 tables.	Children learn the times-tables as 'groups of' but apply their knowledge of commutativity.   *I can use the ×3 table to work out how many keys.* *I can also use the ×3 table to work out how many batteries.*	Children understand how the ×2, ×4 and ×8 tables are related through repeated doubling.  3 x 2 = 6 3 x 4 = 12 3 x 8 = 24	Children understand the relationship between related multiplication and division facts in known times-tables.   *2 × 5 = 10* *5 × 2 = 10* *10 ÷ 5 = 2* *10 ÷ 2 = 5*

Using known facts to multiply 10s, for example 3 × 40	Explore the relationship between known times-tables and multiples of 10 using place value equipment. *Make 4 groups of 3 ones.*  *Make 4 groups of 3 tens.*  *What is the same?* *What is different?*	Understand how unitising 10s supports multiplying by multiples of 10.  *4 groups of 2 ones is 8 ones.* *4 groups of 2 tens is 8 tens.* *4 × 2 = 8* *4 × 20 = 80*	Understand how to use known times-tables to multiply multiples of 10.  *4 × 2 = 8* *4 × 20 = 80*

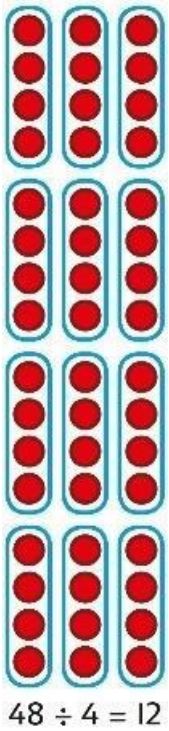
Multiplying a 2-digit number by a 1-digit number	Understand how to link partitioning a 2-digit number with multiplying. *Each person has 23 flowers.* *Each person has 2 tens and 3 ones.*  *There are 3 groups of 2 tens.* *There are 3 groups of 3 ones.* Use place value equipment to model the multiplication context.  *There are 3 groups of 3 ones.*	Use place value to support how partitioning is linked with multiplying by a 2-digit number. *3 × 24 = ?*  *3 × 4 = 12*  *3 × 20 = 60* *60 + 12 = 72* *3 × 24 = 72*	Use addition to complete multiplications of 2-digit numbers by a 1-digit number. *4 × 13 = ?* *4 × 3 = 12* *4 × 10 = 40* *12 + 40 = 52* *4 × 13 = 52*

	There are 3 groups of 2 tens.		
Multiplying a 2-digit number by a 1-digit number, expanded column method	Use place value equipment to model how 10 ones are exchanged for a 10 in some multiplications. *3 × 24 = ?* *3 × 20 = 60* *3 × 4 = 12*  *3 × 24 = 60 + 12* *3 × 24 = 70 + 2* *3 × 24 = 72*	Understand that multiplications may require an exchange of 1s for 10s, and also 10s for 100s. *4 × 23 = ?*  	Children may write calculations in expanded column form, but must understand the link with place value and exchange. Children are encouraged to write the expanded parts of the calculation separately.  *5 × 28 = ?*

		4 × 23 = 92  *5 × 23 = ?* 5 × 3 = 15 *5 × 20 = 100* *5 × 23 = 115*	T O 2 8 × 5 4 0 (5 × 8) 1 0 0 (5 × 20) 1 4 0
Year 3 Division			
Using timetables knowledge to divide	Use knowledge of known times-tables to calculate divisions.	Use knowledge of known times-tables to calculate divisions.	Use knowledge of known times-tables to calculate divisions. *I need to work out 30 shared between 5.*

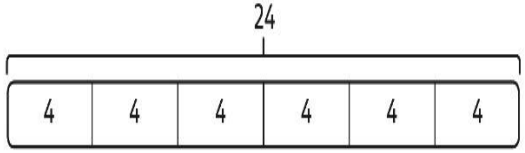


24 divided into groups of 8.
There are 3 groups of 8.

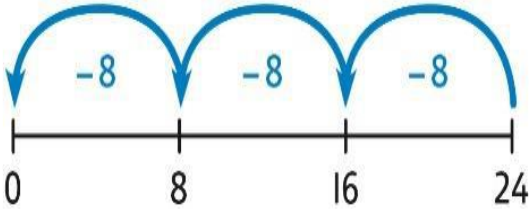


48 divided into groups of 4.
There are 12 groups.
4 × 12 = 48
48 ÷ 4 = 12

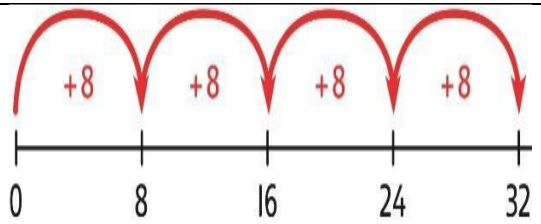
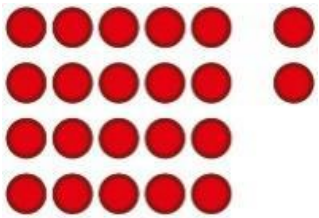
I know that 6 × 5 = 30 so I know that 30 ÷ 5 = 6.
A bar model may represent the relationship between sharing and grouping.

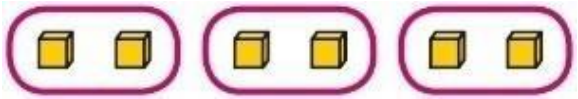
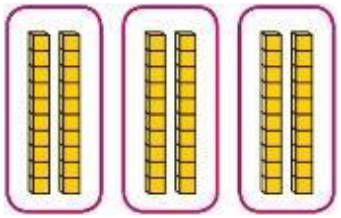
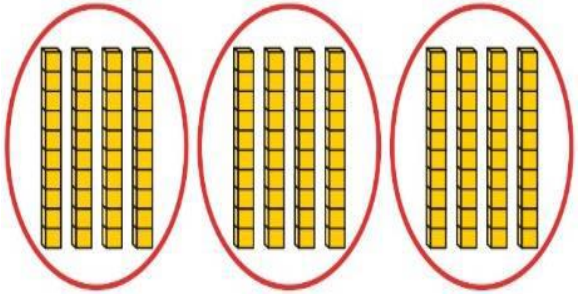


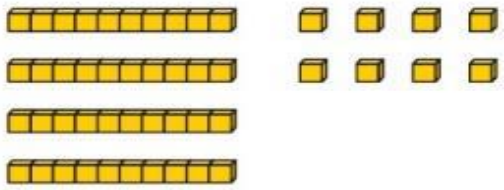
24 ÷ 4 = 6
24 ÷ 6 = 4
Children understand how division is related to both repeated subtraction and repeated addition.



24 ÷ 8 = 3

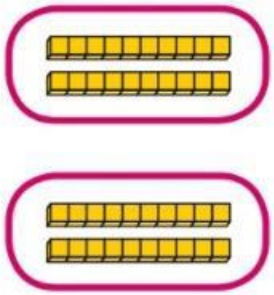
			 *32 ÷ 8 = 4*
Understanding remainders	Use equipment to understand that a remainder occurs when a set of objects cannot be divided equally any further.  *There are 13 sticks in total.* *There are 3 groups of 4, with 1 remainder.*	Use images to explain remainders.  *22 ÷ 5 = 4 remainder 2*	Understand that the remainder is what cannot be shared equally from a set. *22 ÷ 5 = ?* *3 × 5 = 15* *4 × 5 = 20* *5 × 5 = 25 . . . this is larger than 22* *So, 22 ÷ 5 = 4 remainder 2*
Using known facts to divide multiples of 10	Use place value equipment to understand how to divide by unitising.	Divide multiples of 10 by unitising.	Divide multiples of 10 by a single digit using known times-tables. *180 ÷ 3 = ?* *180 is 18 tens.* *18 divided by 3 is 6.*

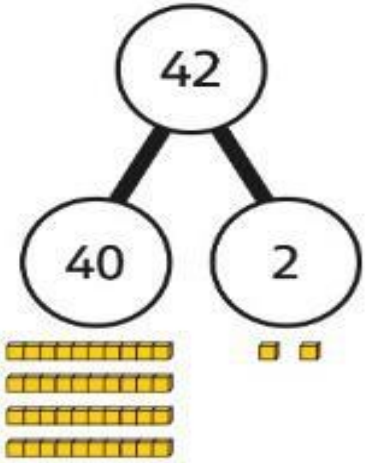
	Make 6 ones divided by 3.  *Now make 6 tens divided by 3.*  *What is the same? What is different?*	 *12 tens shared into 3 equal groups.* *4 tens in each group.*	*18 tens divided by 3 is 6 tens.* *18 ÷ 3 = 6* *180 ÷ 3 = 60*
2-digit number divided by 1-digit number, no remainders	Children explore dividing 2-digit numbers by using place value equipment.	Children explore which partitions support particular divisions.	Children partition a number into 10s and 1s to divide where appropriate.



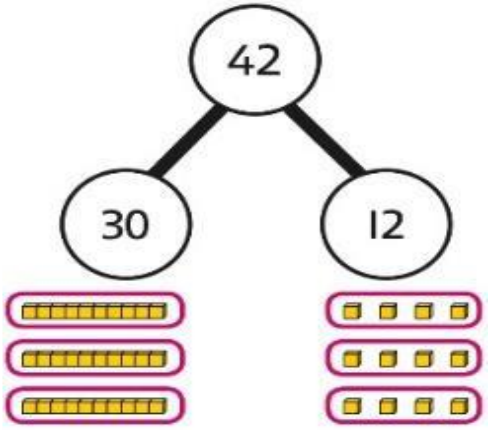
48 ÷ 2 = ?

First divide the 10s.

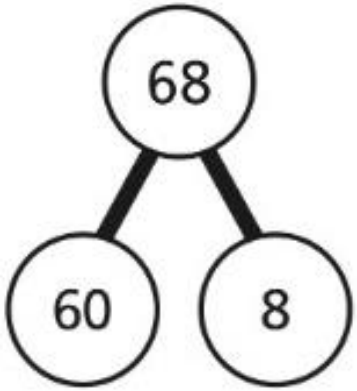




I need to partition 42 differently to divide by 3.



42 = 30 + 12



60 ÷ 2 = 30

8 ÷ 2 = 4

30 + 4 = 34

68 ÷ 2 = 34

Children partition flexibly to divide where appropriate.

42 ÷ 3 = ?

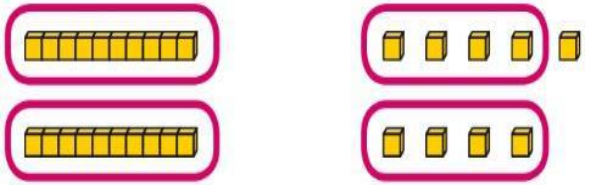
42 = 40 + 2

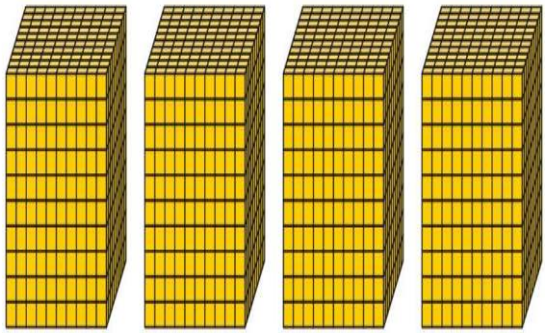
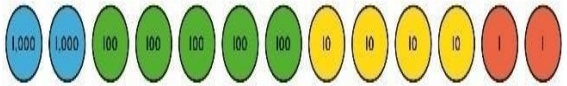
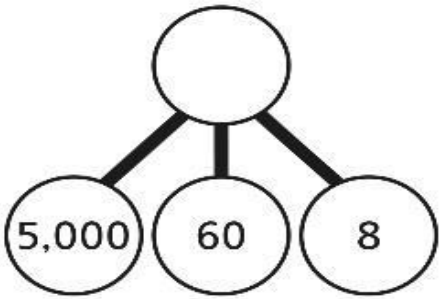
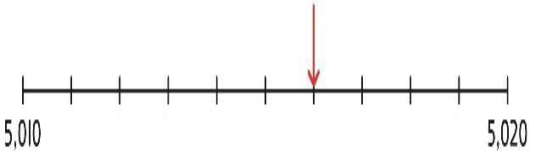
I need to partition 42 differently to divide by 3.

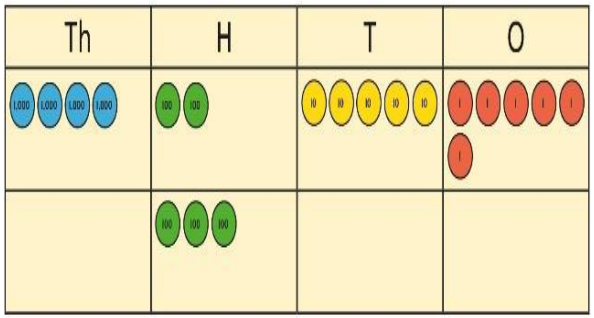
42 = 30 + 12

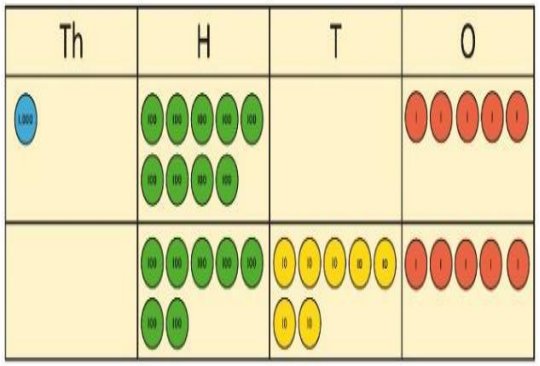
30 ÷ 3 = 10

12 ÷ 3 = 4

	Then divide the 1s. 	*42 ÷ 3 = 14*	*10 + 4 = 14* *42 ÷ 3 = 14*
2-digit number divided by 1-digit number, with remainders	Use place value equipment to understand the concept of remainder. *Make 29 from place value equipment.* *Share it into 2 equal groups.*  *There are two groups of 14 and 1 remainder.*	Use place value equipment to understand the concept of remainder in division. *29 ÷ 2 = ?*  *29 ÷ 2 = 14 remainder 1*	Partition to divide, understanding the remainder in context. *67 children try to make 5 equal lines.* *67 = 50 + 17* *50 ÷ 5 = 10* *17 ÷ 5 = 3 remainder 2* *67 ÷ 5 = 13 remainder 2* *There are 13 children in each line and 2 children left out.*

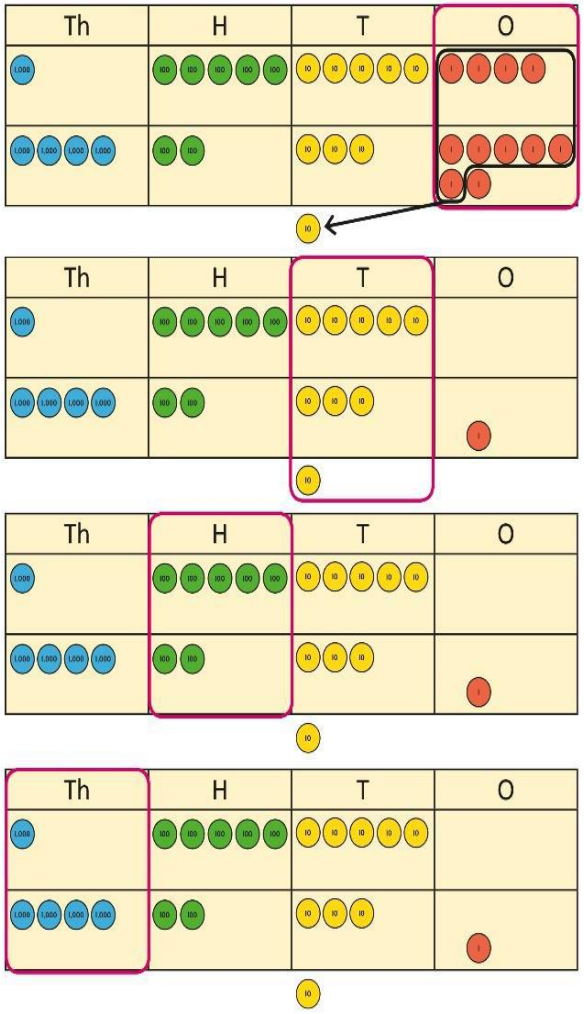
Year 4			
Year 4 addition	Concrete	Pictorial	Abstract
Understanding numbers to 10,000	Use place value equipment to understand the place value of 4-digit numbers.  *4 thousands equal 4,000.* *1 thousand is 10 hundreds.*	Represent numbers using place value counters once children understand the relationship between 1,000s and 100s.  *2,000 + 500 + 40 + 2 = 2,542*	Understand partitioning of 4-digit numbers, including numbers with digits of 0.  *5,000 + 60 + 8 = 5,068* Understand and read 4-digit numbers on a number line. 
Choosing mental	Use unitising and known facts to support mental calculations.	Use unitising and known facts to support mental calculations.	

methods where appropriate	*Make 1,405 from place value equipment.* *Add 2,000.* *Now add the 1,000s.* *1 thousand + 2 thousands = 3 thousands* *1,405 + 2,000 = 3,405*	 *I can add the 100s mentally.* *200 + 300 = 500* *So, 4,256 + 300 = 4,556*	Use unitising and known facts to support mental calculations. *4,256 + 300 = ?* *2 + 3 = 5 200 + 300 = 500* *4,256 + 300 = 4,556*
Column addition with exchange	Use place value equipment on a place value grid to organise thinking. Ensure that children understand how the columns relate to place value and what to do if the numbers are not all 4-digit numbers. *Use equipment to show 1,905 + 775.*	Use place value equipment to model required exchanges.	Use a column method to add, including exchanges.



Why have only three columns been used for the second row? Why is the Thousands box empty?

Which columns will total 10 or more?

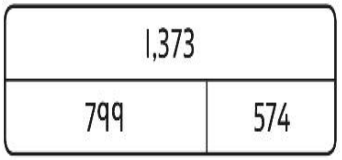
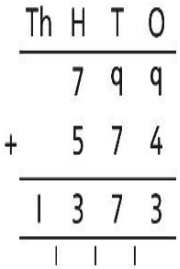
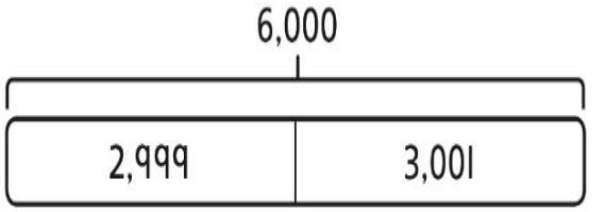
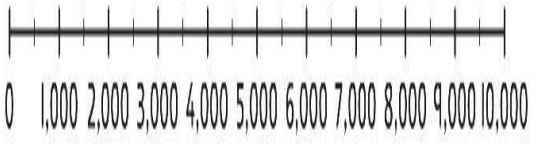


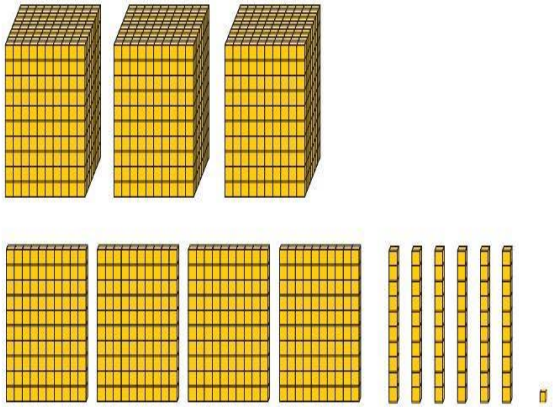
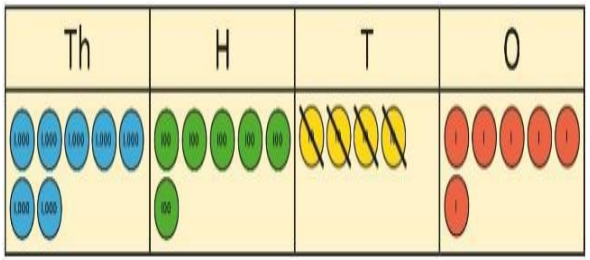
	Th	H	T	O
	1	5	5	4
+	4	2	3	7
				1
			1	

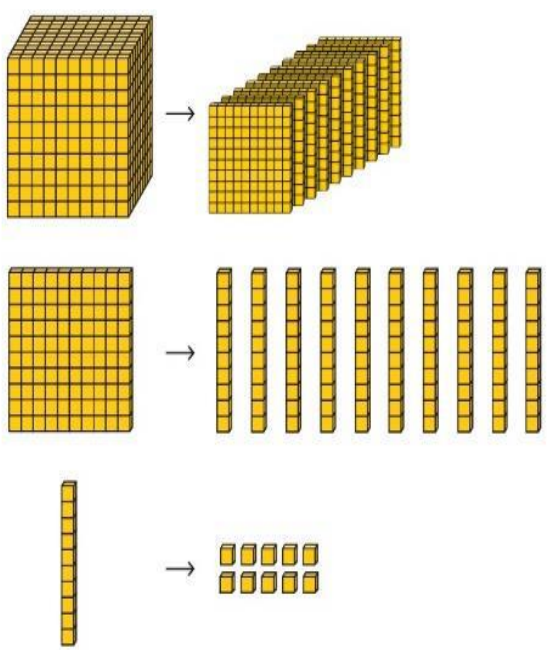
	Th	H	T	O
	1	5	5	4
+	4	2	3	7
			9	1
			1	

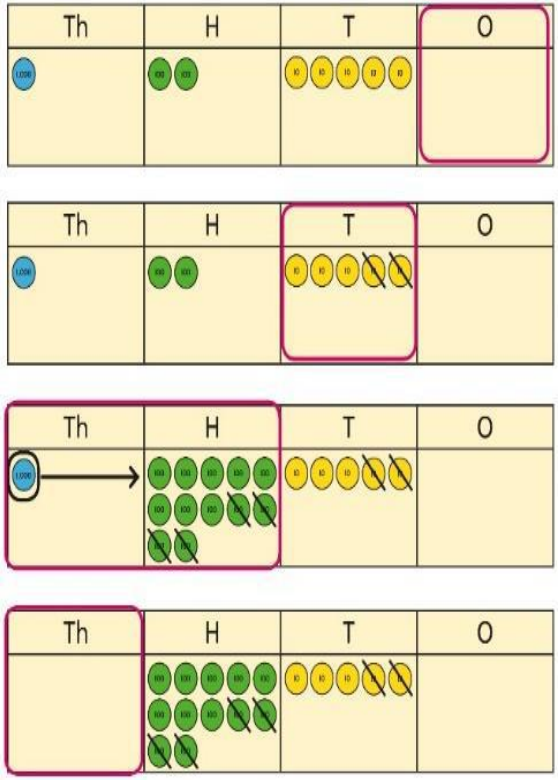
	Th	H	T	O
	1	5	5	4
+	4	2	3	7
		7	9	1
		1		

	Th	H	T	O
	1	5	5	4
+	4	2	3	7
	5	7	9	1
		1		

		Include examples that exchange in more than one column.	Include examples that exchange in more than one column.
Representing additions and checking strategies		Bar models may be used to represent additions in problem contexts, and to justify mental methods where appropriate.   *I chose to work out 574 + 800, then subtract 1.*  *This is equivalent to 3,000 + 3,000.*	Use rounding and estimating on a number line to check the reasonableness of an addition.  *912 + 6,149 = ?* *I used rounding to work out that the answer should be approximately 1,000 + 6,000 = 7,000.*

Year 4 Subtraction			
Choosing mental methods where appropriate	Use place value equipment to justify mental methods.  *What number will be left if we take away 300?*	Use place value grids to support mental methods where appropriate.  *7,646 – 40 = 7,606*	Use knowledge of place value and unitising to subtract mentally where appropriate. *3,501 – 2,000* *3 thousands – 2 thousands = 1 thousand* *3,501 – 2,000 = 1,501*
Column subtraction with exchange	Understand why exchange of a 1,000 for 100s, a 100 for 10s, or a 10 for 1s may be necessary.	Represent place value equipment on a place value grid to subtract, including exchanges where needed.	Use column subtraction, with understanding of the place value of any exchange required.



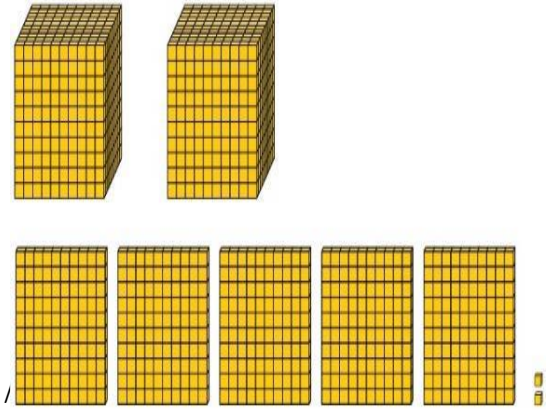
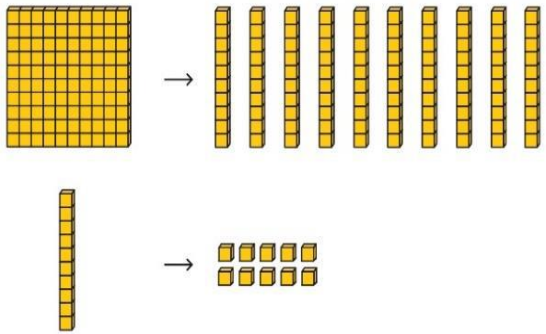
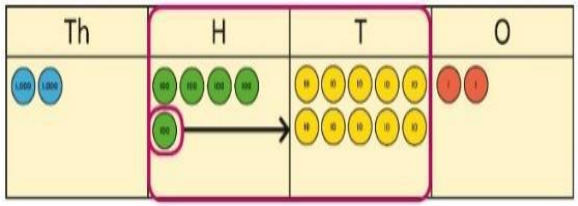
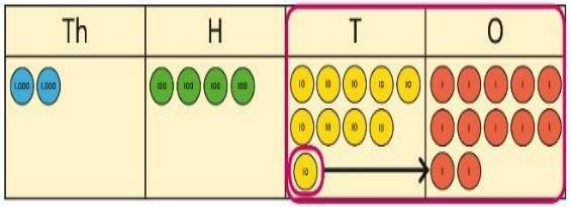
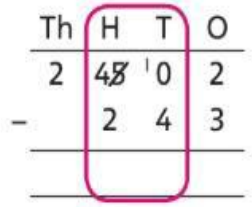
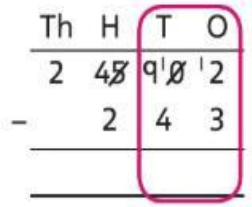
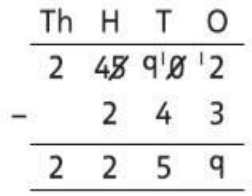


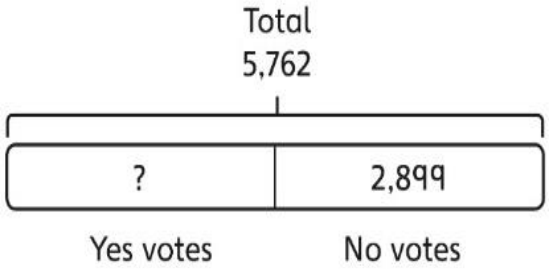
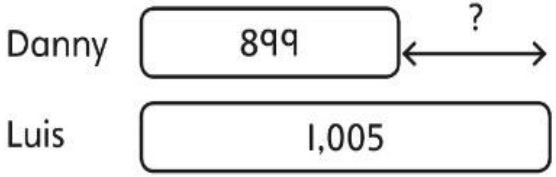
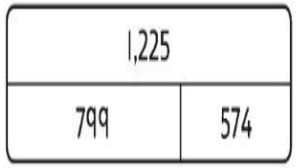
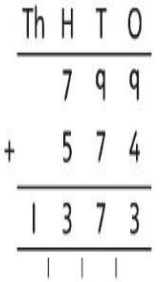
	Th	H	T	O
	1	2	5	0
−		4	2	0
				0

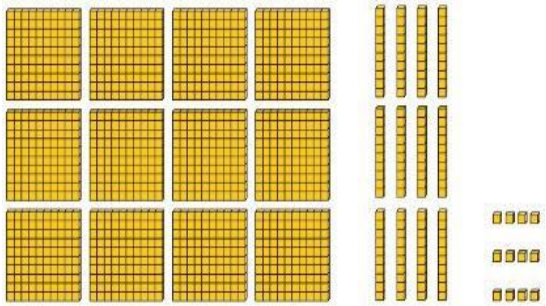
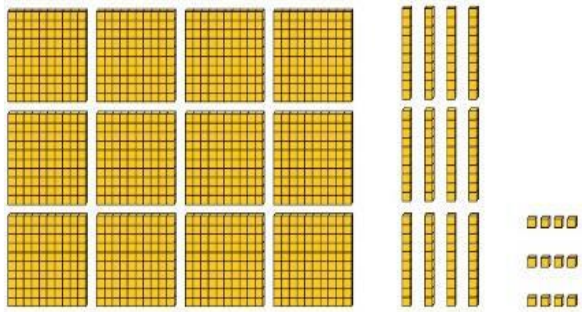
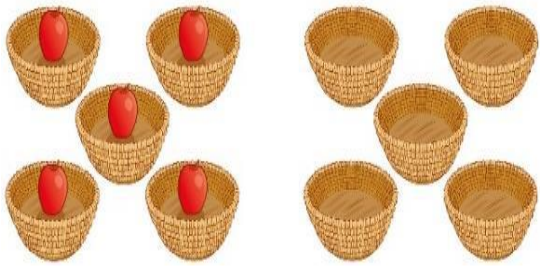
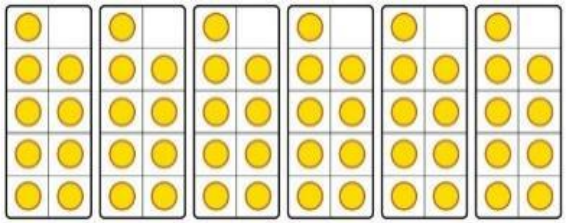
	Th	H	T	O
	1	2	5	0
−		4	2	0
			3	0

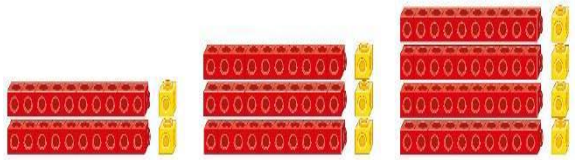
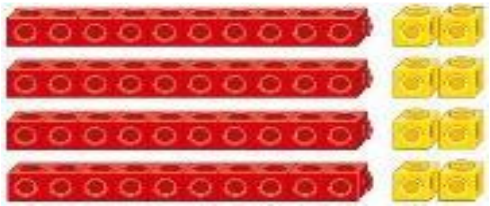
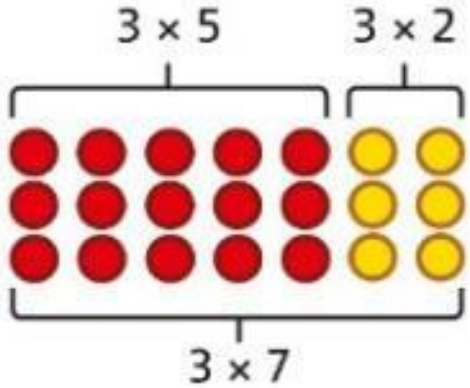
	Th	H	T	O
	✗	¹2	5	0
−		4	2	0
		8	3	0

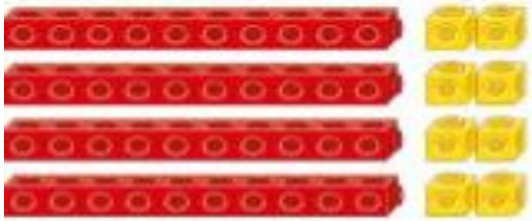
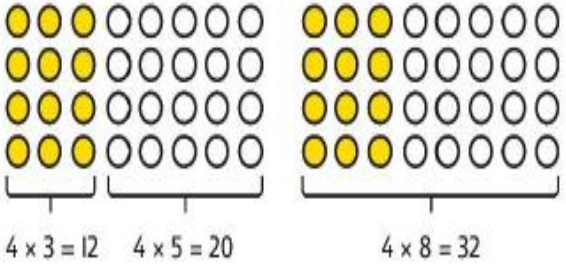
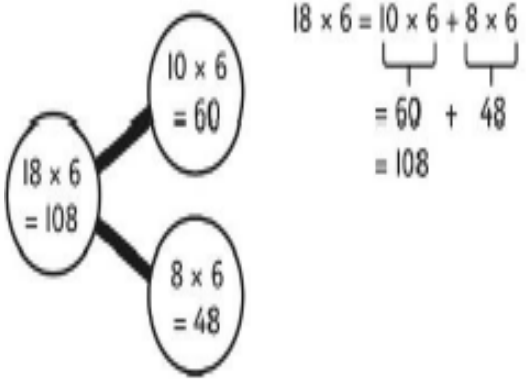
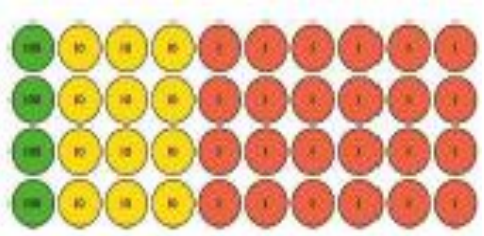
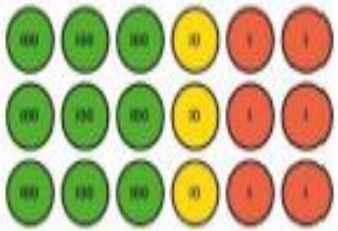
	Th	H	T	O
	✗	¹2	5	0
−		4	2	0
		8	3	0

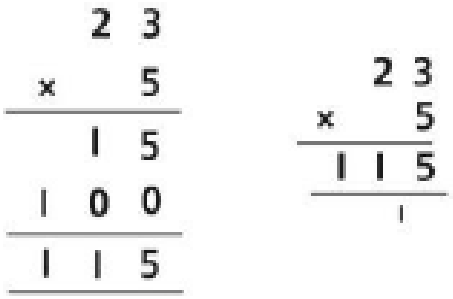
Column subtraction with exchange across more than one column	Understand why two exchanges may be necessary. *2,502 – 243 = ?*  *need to exchange a 10 for some 1s, but there are not any 10s here.* 	Make exchanges across more than one column where there is a zero as a place holder. *2,502 – 243 = ?*  	Make exchanges across more than one column where there is a zero as a place holder. *2,502 – 243 = ?*   

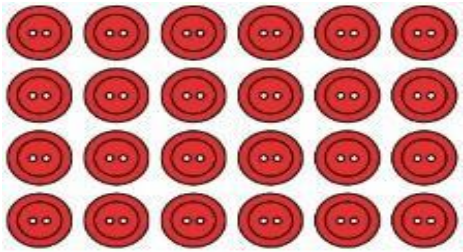
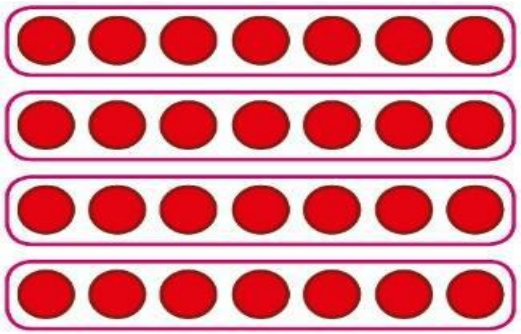
Representing subtractions and checking strategies		Use bar models to represent subtractions where a part needs to be calculated.  *I can work out the total number of Yes votes using 5,762 – 2,899.* Bar models can also represent 'find the difference' as a subtraction problem.  	Use inverse operations to check subtractions. *I calculated 1,225 – 799 = 574.* *I will check by adding the parts.*   *The parts do not add to make 1,225.* *I must have made a mistake.*
Year 4 Multiplication			

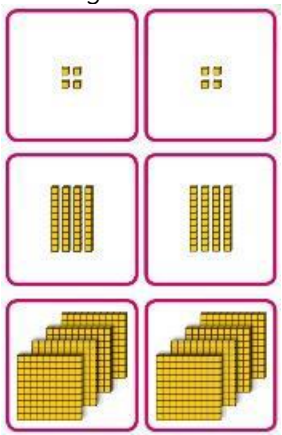
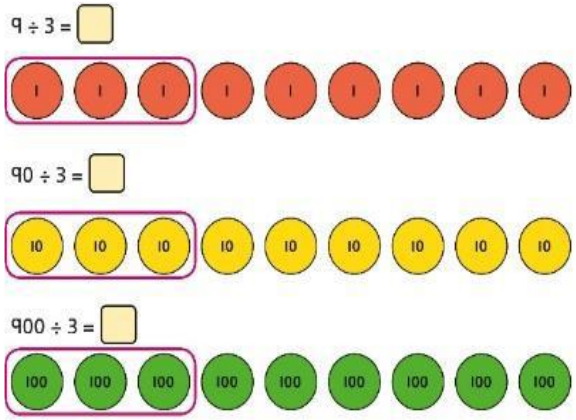
Multiplying by multiples of 10 and 100	Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.  *3 groups of 4 ones is 12 ones.* *3 groups of 4 tens is 12 tens.* *3 groups of 4 hundreds is 12 hundreds.*	Use unitising and place value equipment to understand how to multiply by multiples of 1, 10 and 100.  *3 × 4 = 12* *3 × 40 = 120* *3 × 400 = 1,200*	Use known facts and understanding of place value and commutativity to multiply mentally. *4 × 7 = 28* *4 × 70 = 280* *40 × 7 = 280* *4 × 700 = 2,800* *400 × 7 = 2,800*
Understanding times-tables up to 12 × 12	Understand the special cases of multiplying by 1 and 0.  *5 × 1 = 5* *5 × 0 = 0*	Represent the relationship between the ×9 table and the ×10 table. 	Understand how times-tables relate to counting patterns. Understand links between the ×3 table, ×6 table and ×9 table *5 × 6 is double 5 × 3* ×5 table and ×6 table *I know that 7 × 5 = 35 so I know that 7 × 6 = 35 + 7.* ×5 table and ×7 table

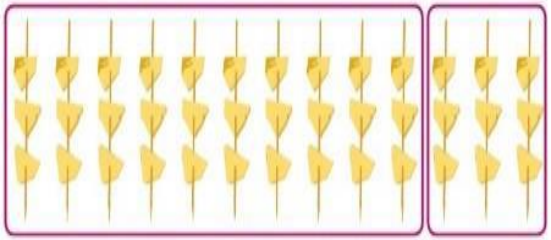
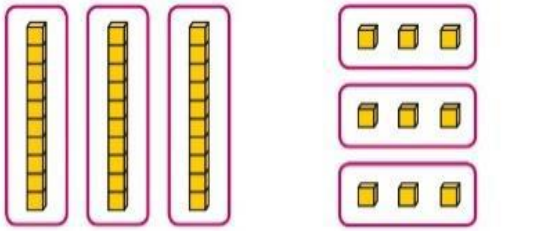
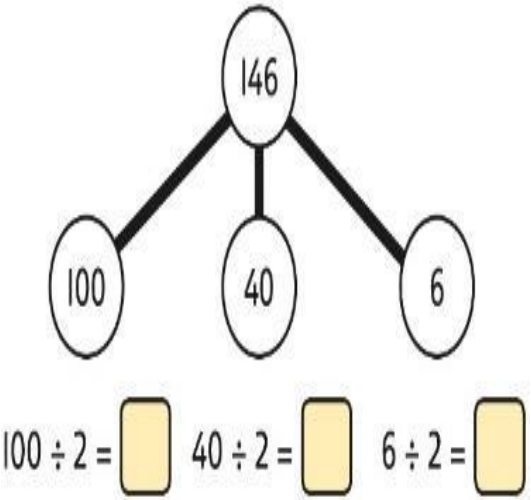
		Represent the ×11 table and ×12 tables in relation to the ×10 table.  *2 × 11 = 20 + 2* *3 × 11 = 30 + 3* *4 × 11 = 40 + 4*  *4 × 12 = 40 + 8*	3X5=3X5+3X2  ×9 table and ×10 table *6 × 10 = 60* *6 × 9 = 60 – 6*
Understanding and using partitioning in multiplication	Make multiplications by partitioning. *4 × 12 is 4 groups of 10 and 4 groups of 2.*	Understand how multiplication and partitioning are related through addition.	Use partitioning to multiply 2-digit numbers by a single digit. *18 × 6 = ?*

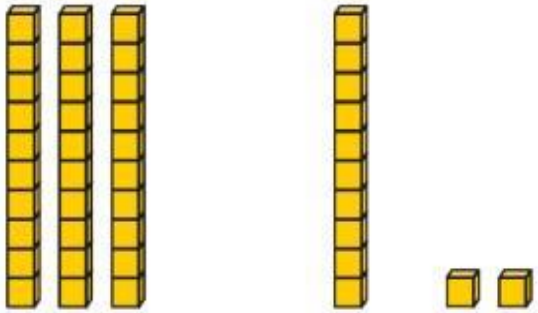
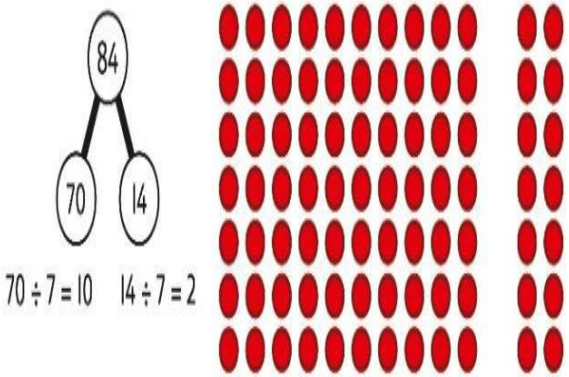
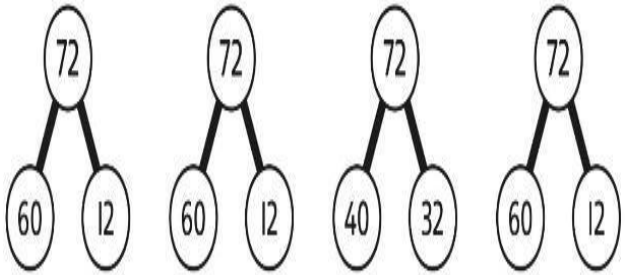
	 *4 × 12 = 40 + 8*	 *4 × 3 = 12* *4 × 5 = 20* *12 + 20 = 32* *4 × 8 = 32*	 *18 × 6 = 10 × 6 + 8 × 6* *= 60 + 48* *= 108*
Column multiplication for 2- and 3-digit numbers multiplied by a single digit	Use place value equipment to make multiplications. *Make 4 × 136 using equipment.* 	Use place value equipment alongside a column method for multiplication of up to 3-digit numbers by a single digit.  3 1 2 x 3 9 3 6	Use the formal column method for up to 3-digit numbers multiplied by a single digit. 3 1 2 x 3 9 3 6

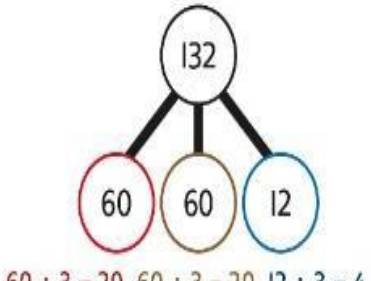
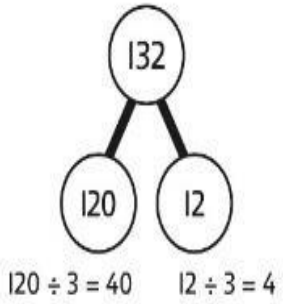
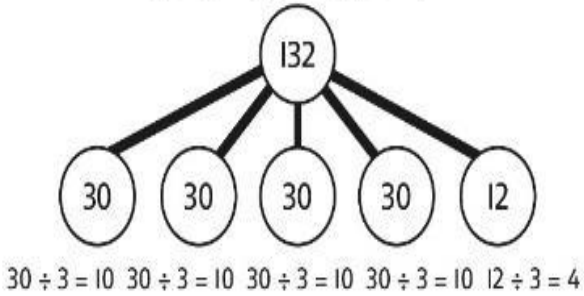
	I can work out how many 1s, 10s and 100s. *There are 4 × 6 ones 24 ones* *There are 4 × 3 tens 12 tens* *There are 4 × 1 hundred 4 hundreds* *24 + 120 + 400 = 544*		Understand how the expanded column method is related to the formal column method and understand how any exchanges are related to place value at each stage of the calculation. 
Multiplying more than two numbers	Represent situations by multiplying three numbers together.  *Each sheet has 2 × 5 stickers.* *There are 3 sheets.*	Understand that commutativity can be used to multiply in different orders.  *2 × 6 × 10 = 120* 12 × 10 = 120	Use knowledge of factors to simplify some multiplications. *24 × 5 = 12 × 2 × 5*

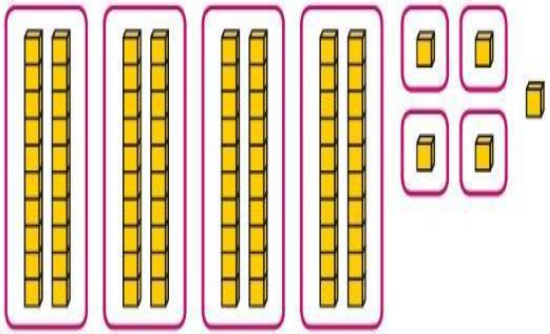
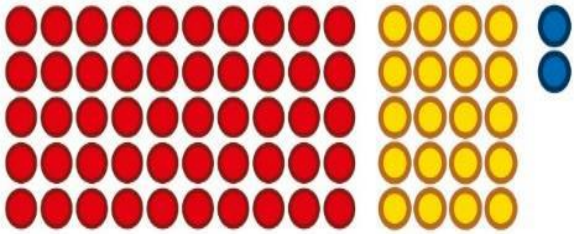
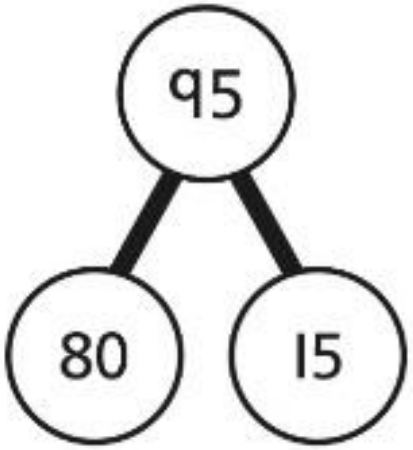
	There are 5 × 2 × 3 stickers in total. 5 × 2 × 3 = 30 10 × 3 = 30	*10 × 6 × 2 = 120* 60 × 2 = 120	12 × 2 × 5 = 12 × 10 = 120 So, 24 × 5 = 120
Year 4 Division			
Understanding the relationship between multiplication and division, including times-tables	Use objects to explore families of multiplication and division facts.  *4 × 6 = 24*	Represent divisions using an array.  28 ÷ 7 = 4	Understand families of related multiplication and division facts. *I know that 5 × 7 = 35* *so I know all these facts:* *5 × 7 = 35* *7 × 5 = 35* *35 = 5 × 7* *35 = 7 × 5*

	24 is 6 groups of 4. *24 is 4 groups of 6.* *24 divided by 6 is 4.* *24 divided by 4 is 6.*		*35 ÷ 5 = 7* *35 ÷ 7 = 5* *7 = 35 ÷ 5* *5 = 35 ÷ 7*
Dividing multiples of 10 and 100 by a single digit	Use place value equipment to understand how to use unitising to divide.  *8 ones divided into 2 equal groups* *4 ones in each group* *8 tens divided into 2 equal groups* *4 tens in each group* *8 hundreds divided into 2 equal groups* *4 hundreds in each group*	Represent divisions using place value equipment.  *9 ÷ 3 = 3* *9 tens divided by 3 is 3 tens.* *9 hundreds divided by 3 is 3 hundreds.*	Use known facts to divide 10s and 100s by a single digit. *15 ÷ 3 = 5* *150 ÷ 3 = 50* *1500 ÷ 3 = 500*

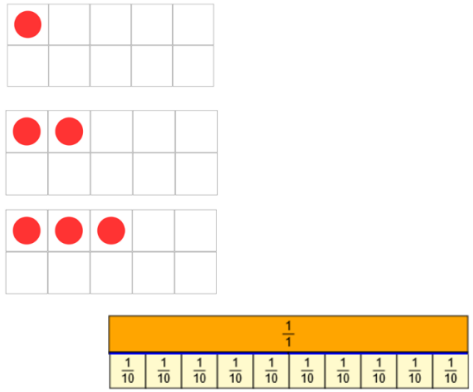
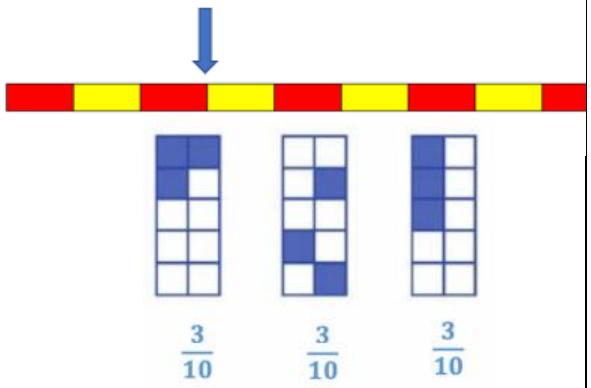
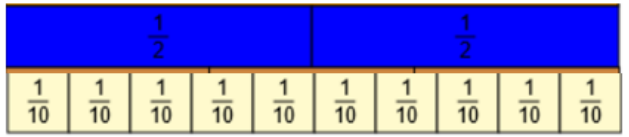
Dividing 2-digit and 3-digit numbers by a single digit by partitioning into 100s, 10s and 1s	Partition into 10s and 1s to divide where appropriate. *39 ÷ 3 = ?*  3 x 10 = 30 3 x 3 = 9 *39 = 30 + 9* *30 ÷ 3 = 10* 9 ÷ 3 = 3 *39 ÷ 3 = 13*	Partition into 100s, 10s and 1s using Base 10 equipment to divide where appropriate. *39 ÷ 3 = ?*  3 groups of 1 ten 3 groups of 3 ones *39 = 30 + 9* *30 ÷ 3 = 10* 9 ÷ 3 = 3 *39 ÷ 3 = 13*	Partition into 100s, 10s and 1s using a part-whole model to divide where appropriate. *142 ÷ 2 = ?*  *100 ÷ 2 = 50* 40 ÷ 2 = 20 6 ÷ 2 = 3 *50 + 20 + 3 = 73* 142 ÷ 2 = 73

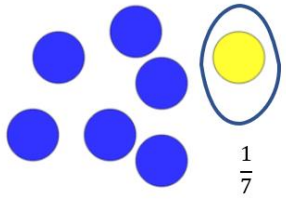
Dividing 2-digit and 3-digit numbers by a single digit, using flexible partitioning	Use place value equipment to explore why different partitions are needed. *42 ÷ 3 = ?* *I will split it into 30 and 12, so that I can divide by 3 more easily.* 	Represent how to partition flexibly where needed. *84 ÷ 7 = ?* *I will partition into 70 and 14 because I am dividing by 7.*  84 ÷ 7 = 12	Make decisions about appropriate partitioning based on the division required.  72 ÷ 2 = 36 72 ÷ 3 = 24 72 ÷ 4 = 18 72 ÷ 6 = 12 Understand that different partitions can be used to complete the same division.

			 132 60 60 12 60 ÷ 3 = 20 60 ÷ 3 = 20 12 ÷ 3 = 4 132 ÷ 3 = 44  132 120 12 120 ÷ 3 = 40 12 ÷ 3 = 4  132 30 30 30 30 12 30 ÷ 3 = 10 30 ÷ 3 = 10 30 ÷ 3 = 10 30 ÷ 3 = 10 12 ÷ 3 = 4

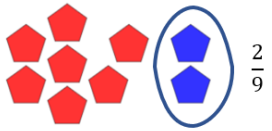
Understanding remainders	Use place value equipment to find remainders. *85 shared into 4 equal groups* *There are 24, and 1 that cannot be shared.* 	Represent the remainder as the part that cannot be shared equally.  *72 ÷ 5 = 14 remainder 2*	Understand how partitioning can reveal remainders of divisions.  *80 ÷ 4 = 20* *12 ÷ 4 = 3* *95 ÷ 4 = 23 remainder 3*

Lower KS2 – Progression in Fractions

Y3 Concrete	Y3 Pictorial	Y3 Abstract
Objective 1: Count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10.		
		
Objective 2: Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators.		

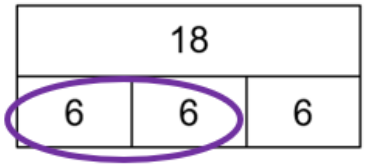


$\frac{1}{7}$

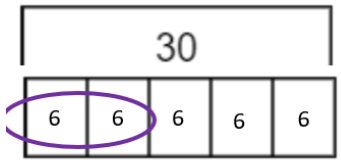


$\frac{2}{9}$

Find $\frac{1}{4}$ of 84



$\frac{2}{3}$ *of* $18 = 12$



$\frac{2}{5}$ *of* $30 = 12$

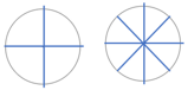
6 × 3 = 18 **18 ÷ 6 = 3** **$\frac{1}{6}$ of 18 = 3**

3 × 6 = 18 **18 ÷ 3 = 6** **$\frac{1}{3}$ of 18 = 6**

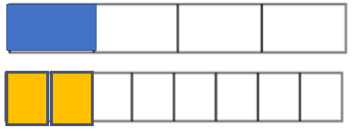
$\frac{1}{4}$ *of* $60 = \frac{1}{3}$ *of* 45

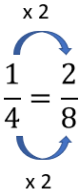
$\frac{2}{5}$ *of* $60 = \frac{1}{2}$ *of* 48

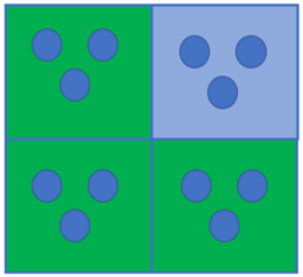
Objective 3: Recognise and show, using diagrams, equivalent fractions with small denominators.



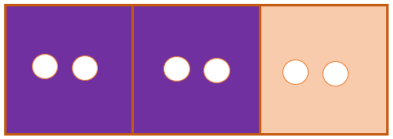
Show quarters and eighths



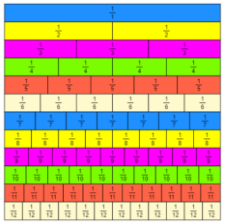




$$\frac{3}{4} = \frac{9}{12}$$



$$\frac{2}{3} = \frac{4}{6}$$



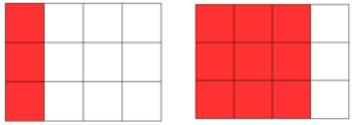
$$\frac{4}{5} \overset{\times 2}{=} \frac{8}{10}$$

$$\frac{3}{6} \overset{\times 5}{=} \frac{15}{30}$$

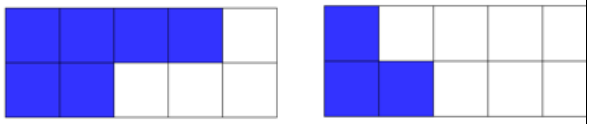
$$\frac{3}{9} \overset{\times ?}{=} \frac{6}{18}$$

Objective 4: Add and Subtract fractions with the same denominator within one whole.

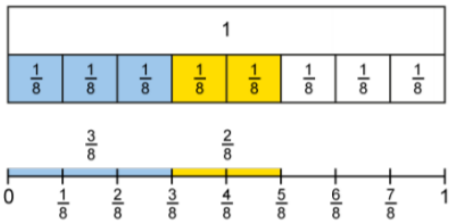
$$\frac{2}{7}+\frac{3}{7}=\frac{5}{7}$$



$$\frac{1}{4}+\frac{3}{4}=\frac{4}{4}$$



$$\frac{6}{10}-\frac{3}{10}=\frac{3}{10}$$



$\frac{3}{8}+\frac{2}{8}=\frac{5}{8}$ $\quad$ $\frac{5}{8}-\frac{2}{8}=\frac{3}{8}$

Write three fraction equations for this model.

Answer

$$\frac{3}{8}+\frac{4}{8}=$$

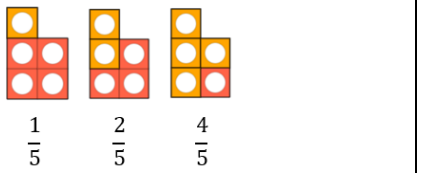
$$\frac{5}{7}-\frac{2}{7}=$$

True or false?

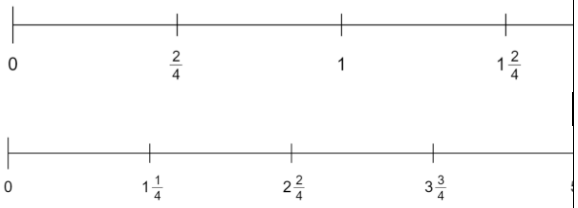
$$\frac{5}{6}+\frac{2}{6}=\frac{7}{12}$$

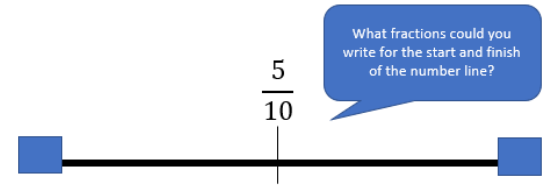
$$\frac{13}{20}-\frac{3}{20}=\frac{1}{2}$$

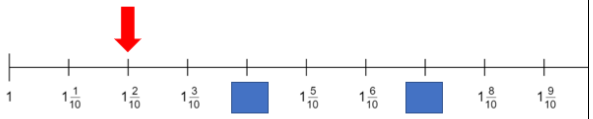
Objective 5: Compare and order unit fractions, and fractions with the same denominators.



$\frac{1}{5}$ $\frac{2}{5}$ $\frac{4}{5}$





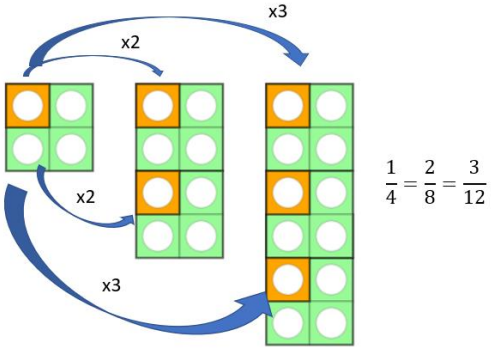
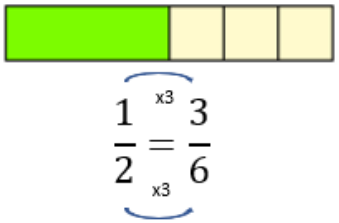
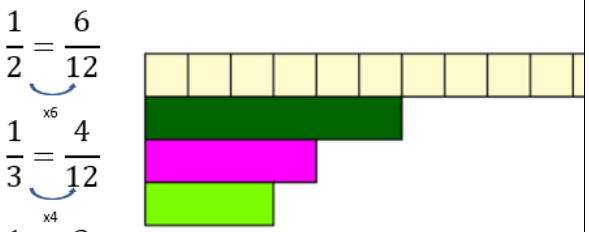
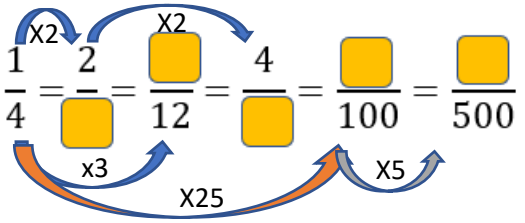
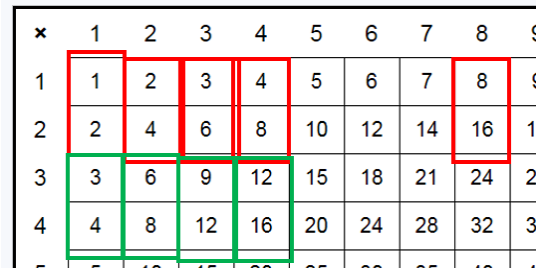
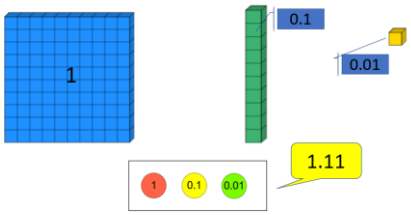
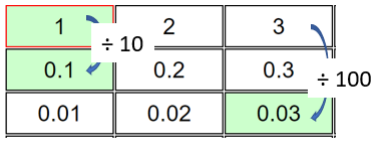
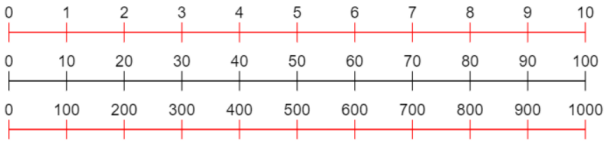


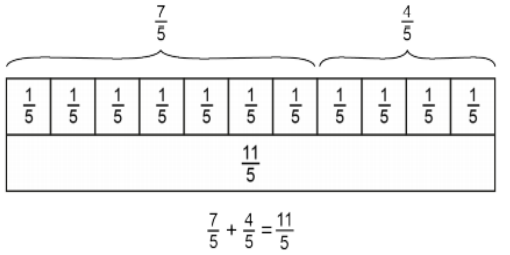
Put <, > or = in the circle to make this statements true.

$\frac{6}{7} \bigcirc \frac{2}{7}$

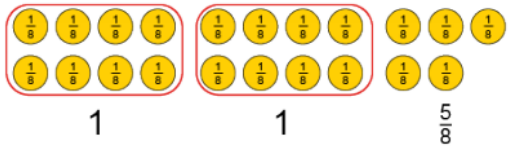
$\frac{5}{10} \bigcirc \frac{8}{16}$

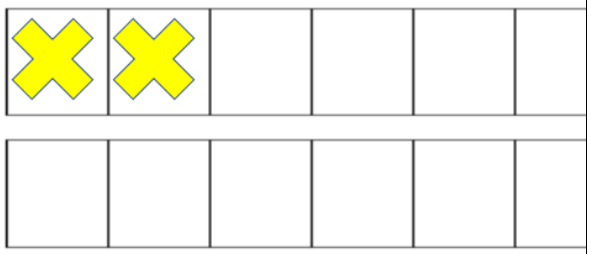
$\frac{6}{15} \bigcirc \frac{3}{15}$

Y4 Concrete	Y4 Pictorial	Y4 Abstract
Objective 1: Recognise and show, using diagrams, families of common equivalent fractions.		
 $\frac{1}{4}=\frac{2}{8}=\frac{3}{12}$	 $\frac{1}{2}\overset{\times 3}{=}\frac{3}{6}$ (×3 below)  $\frac{1}{2}\overset{\times 6}{=}\frac{6}{12}$ $\frac{1}{3}\overset{\times 4}{=}\frac{4}{12}$ $\frac{1}{4}\overset{\times 3}{=}\frac{3}{12}$	 $\frac{1}{4}=\frac{2}{\square}=\frac{\square}{12}=\frac{4}{\square}=\frac{\square}{100}=\frac{\square}{500}$ 
Objective 2: Count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten.		
		
Objective 3: Add and subtract fractions with the same denominator.		



$\frac{7}{5}+\frac{4}{5}=\frac{11}{5}$





$2-\frac{2}{6}=\frac{10}{6}$

$\frac{7}{8}+\frac{3}{8}=$

$4+\frac{6}{8}=$

$3-\frac{5}{6}=$

$\frac{7}{5}+\frac{4}{5}=\frac{11}{5}$

$3\frac{7}{8}-\frac{2}{8}=3\frac{5}{8}$

$7\frac{2}{5}+\frac{4}{5}=8\frac{1}{5}$

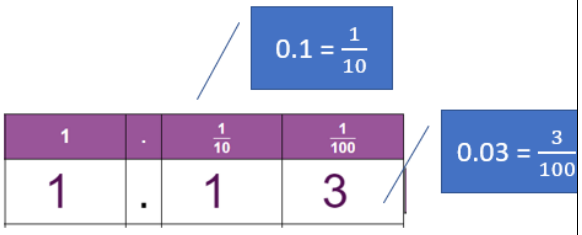
$8\frac{1}{5}-\frac{4}{5}=7\frac{2}{5}$

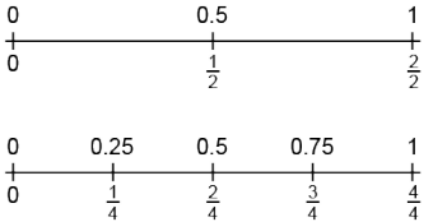
Draw models to prove these statements are true

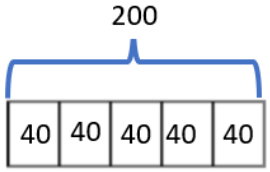
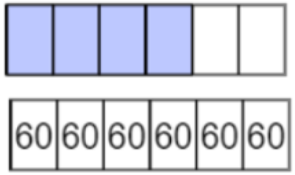
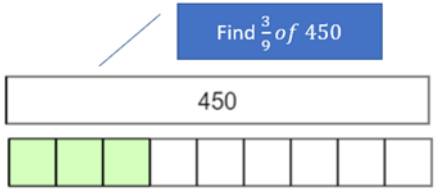
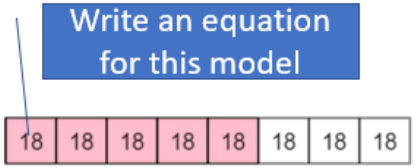
Objective 4: Recognise and write decimal equivalents of any number of tenths or hundredths.

Unit fraction	Decimal fraction
$\frac{1}{2}$	0.5
$\frac{1}{4}$	0.25
$\frac{1}{5}$	0.2
$\frac{1}{10}$	0.1

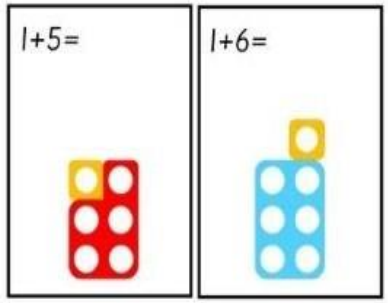
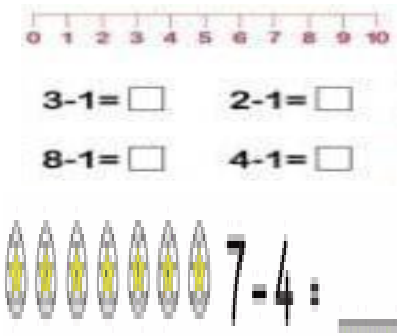
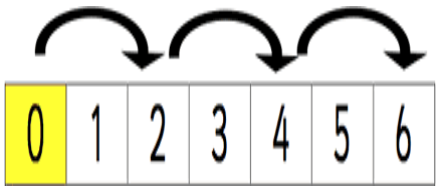
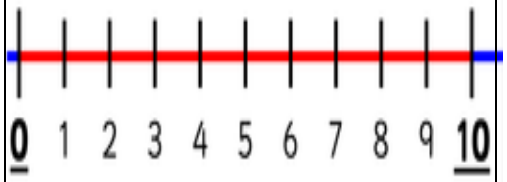
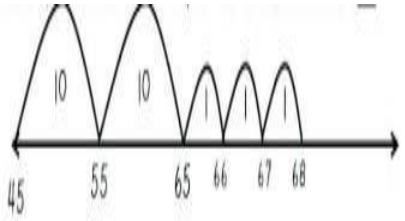
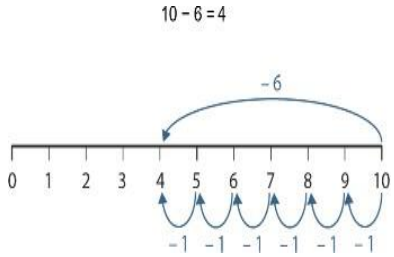
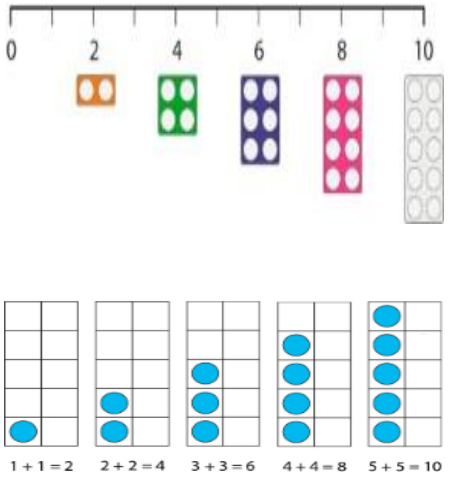
$\frac{1}{4}=\frac{25}{100}=0.25$





Objective 5: Solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number		
 $\frac{3}{5}$ *of* 200 = 120 $\frac{4}{6}$ *of* 360 = 240 	 	Judy and Tilly each have a 1m ribbon. Judy cuts her ribbo equal parts and uses 1 of them to make a hair tie. Tilly cu ribbon into 10 equal parts and uses 3 of them to make a Have Judy and Tilly used the same amount of ribbon? Ex answer. Petrol cans hold 2 litres. Tony uses $\frac{3}{4}$ of a 2 litre can of petrol in his lawn mower. He then uses $\frac{1}{2}$ of another can with $\frac{6}{8}$ remaining of the 2 litres. How much petrol did he use altogether?

Standard Written Method

	Addition	Subtraction	Multiplication	Division
Reception				
Year 1				

Year 2	59 ^{1}43+ 102	^{6}7^{1}3 49- 24	8 x 5 = 40	35 ÷ 5 = 7
Year 3	523 $_{1}$393+ 916	^{4}$_{1}$523 393- 130	59 6x 54 (6x9) 300 (6x50) 354	$\begin{array}{r} 4 \\ 8\overline{)32} \end{array}$
Year 4	1,312 3,$_{1}$$_{1}$094+ 4,406	^{1}^{1}6,273 1,093- 5,180	159 16x954 $_{1}$$_{1}$1,590+ 2,544	$\begin{array}{r} 135 \\ 7\overline{)945} \end{array}$

Year 5	13,123 30,943+ 44,066	62,743 10,923- 51,820	2259 6x 54 300 1,200 12,000+ 13,554	279 r 5 6)1679
Year 6	613,123 1310,943+ 744,066	6112,1743 100,923- 511,820	2259 46x 13,554 901,360+ 103,914	0389.739 23|8964 69- 206 184- 0224 207- 0170 161- 0090 69- 210 207- 003 23 46 69 92 115 138 161 184 207